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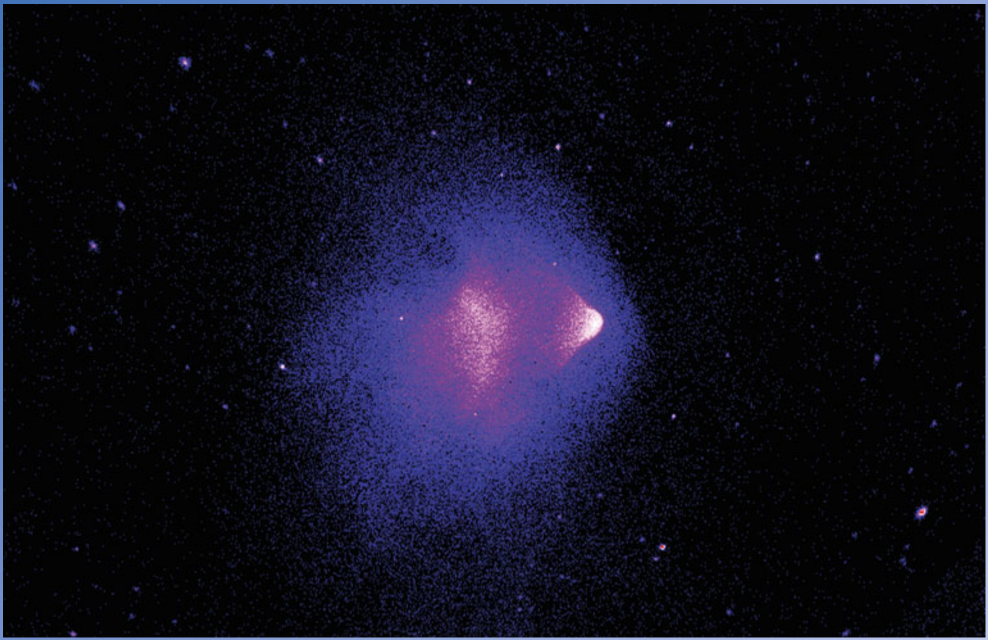
Multi-scale Structure Formation and Dynamics in Cosmic Plasmas



Andre Balogh · Andrei Bykov
Jonathan Eastwood · Jelle Kaastra *Editors*

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INTERNATIONAL SPACE SCIENCE INSTITUTE

Space Sciences Series of ISSI

Volume 51

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Editors

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Previously published in *Space Science Reviews* Volume 188, Issues 1–4, 2015

Springer

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ISSN 1385-7525 Space Sciences Series of ISSI
ISBN 978-1-4939-3546-8 ISBN 978-1-4939-3547-5 (eBook)
DOI 10.1007/978-1-4939-3547-5

Library of Congress Control Number: 2016931277

Springer New York Heidelberg Dordrecht London

Cover Image: X-ray image of the cluster of galaxies 1E0657-56 (the Bullet cluster) made with Chandra X-ray observatory (NASA). The Bullet cluster is a large cosmological plasma structure of a scale size about a megaparsec, which was formed after the collision of two large clusters of galaxies dominated by dark matter. For more details see the chapter Structures and Components in Galaxy Clusters in the book. Credit: M. Markevitch (NASA GSFC)

Printed on acid-free paper

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DOI 10.1007/978-1-4939-3547-5_1
Reprinted from *Space Science Reviews* Journal, DOI 10.1007/s11214-015-0140-4

Multi-scale Structure Formation and Dynamics in Cosmic Plasmas

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Received: 12 February 2015 / Accepted: 13 February 2015 / Published online: 3 March 2015

1 Plasma Structures: From the Earth Magnetosphere to Clusters of Galaxies

Being statistically homogeneous on cosmological scales the Universe is demonstrating a very rich variety of structures and components at smaller scales. Gravity is the source of energy and the driving force for formation of the large scale structure (LSS), clusters of galaxies, galaxies and stars, while the electromagnetic fields are drastically important for the microphysics of the momentum and energy transfer in the baryonic matter and radiation. The Workshop "Multi-scale structure formation and dynamics in cosmic plasmas" which was held at International Space Science Institute in April 2013 was devoted to a broad discussion of different aspects of formation, dynamics, and observational appearance

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of plasma structures at different scales ranging from LSS to the Earth's magnetosphere. The present book is composed from the reviews, which are based on the discussions at the ISSI Workshop. It contains review papers on the basic processes of structure formation in cosmic plasmas starting from electric currents, which produce magnetic structures in planet magnetospheres, stellar winds, and relativistic plasma outflows like pulsar wind nebulae and Active Galactic Nuclei jets. The important role of the helicity concept on the structure formation and evolution of the large scale magnetic fields in highly conductive cosmic plasmas is emphasized in the book. Microscopic dynamics of plasma flows and magnetic fields was discussed in depth in Space Sciences Series of ISSI, Volume 47 "Microphysics of Cosmic Plasmas", originally published as Space Science Reviews (Balogh et al. 2013), and thus, it is highly recommended to refer to that volume while reading the present volume. Cosmological aspects of plasma structures are reviewed within a discussion of large-scale structure formation from the first non-linear objects to massive galaxy clusters, which is followed by a review of observations and current models of structures and components in galaxy clusters. Supernova remnants interacting with molecular clouds are among the most important ingredients of the global galactic ecology with a profound effect on the phenomena related to star formation. The multi-wavelength view from the radio to gamma-rays with modern high resolution telescopes revealed a beautiful and highly informative picture of both coherent and chaotic plasma structures tightly connected by strong mutual influence. The same plasma processes are likely to control the structure and dynamics of Earth's magnetosphere where detailed direct satellite observations are available. The properties of magnetic field fluctuations and structures in the outer solar atmosphere and Earth's magneto-tail, which have direct implications for the general problem of structure formation in hot plasmas, are discussed in depth in the volume.

This volume is aimed at graduate students and researchers active in the areas of astrophysics and space science.

Acknowledgements The Editors are greatly indebted to all the participants of the Workshop held in ISSI Bern on 15 to 19 April 2013 who brought their broad range of expertise and interest in the astrophysics of plasmas to discuss the vast range of scales of plasma structures in the Universe and how the study of their formation and dynamics can illuminate processes at the different scales. We thank the staff of ISSI for their dedicated support: Prof. Rafael Rodrigo, Executive Director, and his colleagues Prof. Rudolf von Steiger, Jennifer Fankhauser, Andrea Fischer, Saliba Saliba and Sylvia Wenger. The resulting collection of review papers was the outcome of the exchanges and fruitful collaboration among the participants; we thank them for their successful efforts to integrate the lessons learned in the different topics, as the reviews in the volume testify. Thanks are also due to the reviewers of the papers; in all cases the reviews were thorough and constructive and the volume bears witness to their contribution. Finally the Editors thank the staff of Space Science Reviews, as well as the production staff for their patience on occasion and for an excellently produced volume.

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A. Balogh, A. Bykov, P. Cargill, R. Dendy, T. Dudok de Wit, J. Raymond, Microphysics of cosmic plasmas: background, motivation and objectives. Space Sci. Rev. **178**, 77–80 (2013). doi:10.1007/s11214-013-0027-1

DOI 10.1007/978-1-4939-3547-5_2
Reprinted from *Space Science Reviews* Journal, DOI 10.1007/s11214-014-0041-y

Electric Current Circuits in Astrophysics

Jan Kuijpers · Harald U. Frey · Lyndsay Fletcher

Received: 27 October 2013 / Accepted: 26 February 2014 / Published online: 5 June 2014

Abstract Cosmic magnetic structures have in common that they are anchored in a dynamo, that an external driver converts kinetic energy into internal magnetic energy, that this magnetic energy is transported as Poynting flux across the magnetically dominated structure, and that the magnetic energy is released in the form of particle acceleration, heating, bulk motion, MHD waves, and radiation. The investigation of the electric current system is particularly illuminating as to the course of events and the physics involved. We demonstrate this for the radio pulsar wind, the solar flare, and terrestrial magnetic storms.

Keywords Cosmic magnetism · Electric circuits · Radio pulsar winds · Solar flares · Terrestrial magnetic storms

1 Introduction

Magnetic field structures in the cosmos occur on many scales, spanning a range of over 15 decades in spatial dimension, from extragalactic winds and jets down to the terrestrial magnetosphere. Yet in all these objects the properties of magnetic fields lead to a very similar, multi-scale, spatial and temporal structure. Magnetic fields originate in electric currents, as described by Maxwell's equations. They have energy, pressure, and tension, as quantified by their energy-momentum (stress) tensor. They exert a force on ionized matter, as

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expressed by the Lorentz force. Finally, their equivalent mass is small when compared to the mass-energy of ambient matter.

These basic properties lead to a common appearance in a variety of objects which can be understood as follows. Since there are no magnetic charges, magnetic fields are not neutralized and they extend over large stretches of space and time. The Lorentz force which keeps ionized matter and magnetic fields together allows gravitation to anchor a magnetic field in a condensed ionized object. Since there gas and dynamic pressures dominate over the magnetic pressure, magnetic fields tend to be amplified by differential motions such as occurs in a stellar convection zone, a differentially rotating accretion disk, binary motion, or a stellar wind around a planet. Expressed in circuit language, a voltage source is set up by fluid motions which drives a current and forms a dynamo in which magnetic field is amplified at the expense of kinetic energy. Next, the small equivalent mass of the magnetic fields makes them buoyant, and as ionized matter slides easily along magnetic fields, they pop up out of the dense dynamo into their dilute environs. Whereas this central domain is dominated by fluid pressure the environs are dilute so that there the magnetic pressure dominates. The tension of the magnetic field then allows for transport of angular momentum from the central body along the field outward into a magnetized atmosphere such as a corona or magnetosphere. As more and more magnetic flux rises into the corona, and/or differential motions at the foot-points of magnetic structures continue to send electric currents and associated Poynting flux into the corona the geometries of these nearly force-free structures adapt, expand and generally lead to the appearance of thin sheets of concentrated electric currents. The same process occurs in the terrestrial magnetosphere but now the Poynting flux is going inward toward the central body. Finally, the magnetic structure in the envelope becomes unstable and the deposited energy is released in a process equivalent to an electromotor or Joule heating but now in a multitude of small 'non-force-free' regions created by the currents themselves, often together in explosions, such as storms, (nano-)flares, ejected plasmoids, jets, but also in a more steady fashion, such as super-fast winds.

The description of the formation and dynamics of complicated magnetic structures in terms of simplified electric current circuits in a variety of objects elucidates the fundamental physics by demanding consistency, and distinguishing cause and effect. Also, it allows for a unified answer to a number of pertinent questions:

– *Current Distribution*: what is the voltage source, how does the current close, which domains can be considered frozen-in, where (and when) is the effective resistivity located?
– *Angular Momentum Transport*: where are the balancing (decelerating and accelerating) torques?
– *Energy Transport*: what is the relative importance of Poynting flux versus kinetic energy flux?
– *Energy Conversion*: what is the nature of the effective resistivity (Lorentz force, reconnection, shocks), and what is the energy partitioning, i.e. the relative importance of gas heating, particle acceleration, bulk flow, and MHD waves?

For this review we have chosen to zoom in on the magnetosphere of the *radio pulsar*, on the *solar flare*, and on *terrestrial aurorae and magnetic storms*. We will point out parallels and similarities in the dynamics of the multi-scale magnetic structures by considering the relevant electric circuits. Many of the same questions (and answers) that are addressed below are relevant to other objects as well, such as extragalactic jets, gamma ray bursts, spinning black holes, and planetary magnetospheres.

Equations will be given in Gaussian units throughout to allow simple comparisons to be made while numerical values are in a variety of units reflecting their usage in the fields they come from.

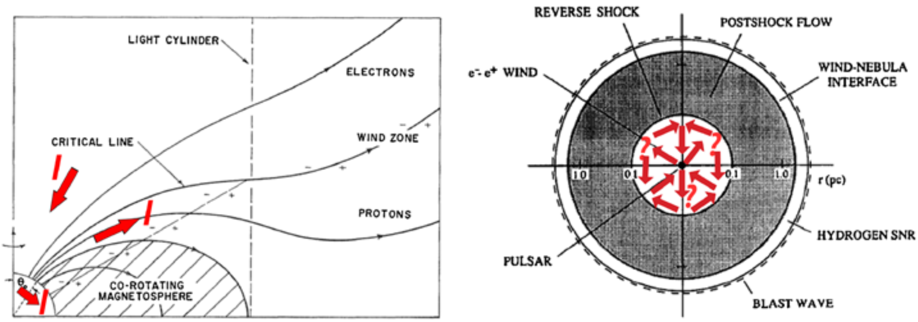

Fig. 1 *Left*: The Goldreich-Julian picture of the pulsar magnetosphere for an aligned magnetic rotator (Goldreich and Julian 1969). The potential difference across the polar cap drives a current I, which enters the star along polar field lines, crosses the stellar surface layers, leaves the star along field lines on the other side of the 'critical' field line, and is supposed to close somewhere in the wind zone. Note that the incoming/outgoing current is carried here by respectively outgoing electrons/outgoing protons, but could equally well be carried by a surplus of outgoing electrons/outgoing positrons. Adaptation from Goldreich and Julian (1969), reproduced by permission of the AAS. *Right*: Sketch of the closure of the current system in the Crab wind, which has to take place before the reverse shock at about 0.1 pc

2 Electric Circuit of the Pulsar Wind

The pulsar wind presents an important class of Poynting flux-dominated outflows. These are dominated by electromagnetic rather than kinetic energy. Here we want to study the closed electric current system, which involves regions both very near to the pulsar and very far away in the pulsar wind, a problem for which no general agreement exists as to its solution. The relative importance of the magnetic and kinetic energy flows is conveniently written as the magnetization σ:

$$\sigma \equiv \frac{\text{Poynting flux}}{\text{kinetic energy flux}} = \left\{ \frac{B_{tor}^2/4\pi}{\rho v_{pol}^2/2}, \frac{B_{tor}^2/4\pi}{\Gamma \rho c^2} \right\} > 1, \tag{1}$$

where we have included the contribution only from the toroidal (i.e. transverse to the radial direction) magnetic field B_{tor} and neglected the poloidal (i.e. in the meridional plane) magnetic field B_{pol} since the latter falls off faster with distance than the former in a steady wind. v_{pol} is the poloidal component of the wind velocity, ρ the wind matter density in the observer frame, and Γ the Lorentz factor of the wind. The first term inside curly brackets applies to non-relativistic and the second to relativistic winds. Further, it is assumed that either the ideal MHD condition applies in the wind:

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B}/c, \tag{2}$$

or that the wind is nearly a vacuum outflow in which case the condition

$$\mathbf{E} = -\mathbf{c} \times \mathbf{B}/c \tag{3}$$

takes over smoothly from the ideal MHD condition. Here $\mathbf{B}$ is the magnetic induction (magnetic field), $\mathbf{v}$ the fluid velocity, and c the speed of light.

The rotating magnetized star which forms the pulsar is a so-called 'unipolar inductor' (Goldreich and Julian 1969). The rotation of the magnetized conductor creates a potential drop across the moving field lines from the magnetic pole towards the equator (see Fig. 1, Left). This voltage drop appears along the field lines between the star and infinity. The reaction of the star to this strong field-aligned potential drop is that, under certain conditions, an electric current is set up (see Fig. 1, Left). Charged particles are drawn out of the crust and accelerated to such high energies that a dense wind of electron-positron pairs leaves the star and provides for the electric currents. The magnitude of the electric current density near the stellar surface is expected to be of order

$$j_{GJ}(r_\star) \equiv \tau_{GJ}(r_\star)c, \tag{4}$$

where the Goldreich-Julian (GJ) charge density is defined as

$$\tau_{GJ}(r) \equiv -\frac{\mathbf{\Omega}_\star \cdot \mathbf{B}_0(\mathbf{r})}{2\pi c}. \tag{5}$$

$\mathbf{B}_0(\mathbf{r})$ is the background magnetic field at position $\mathbf{r}$, $\mathbf{\Omega}_\star$ is the stellar rotation vector, and $\mathbf{r}_\star$ the stellar radius. The GJ charge density just provides for a purely transverse electric field and a corresponding $\mathbf{E} \times \mathbf{B}$-drift which causes the (ideal) plasma to co-rotate with the star at the angular speed $\mathbf{\Omega}_\star$ (2). As a result, when the charge density is equal to the local GJ density everywhere, the parallel electric field vanishes. This is the situation on the 'closed' field lines which are located near the star.

On the open field lines the speed of the charges is assumed to be the speed of light since the wind is expected to be relativistic from the beginning (4). Things are complicated here because a strictly steady state pair creation is not possible. It is clear that the *parallel* electric field momentarily vanishes as soon as the charge density reaches the value τ_{GJ}. However, to produce this GJ-density one needs a very strong parallel electric field to exist. Actually, the strong time-variations within a single radio pulse are believed to mirror the temporal process of pair creation. For our purpose we assume a steady-state relativistic wind to exist in the average sense. This is the reason for the appearance of the GJ density in (4). The current (and the much denser wind) only exist on the so-called 'open' field lines, the field lines which connect to infinity (see Fig. 1, Left). For a steady current the incoming current (poleward for the aligned rotator of Fig. 1, Left) must be balanced by the outgoing current (equatorward). This defines a critical field line in the wind (Fig. 1, Left) separating the two parts of the current. Note, however, that in contrast to Fig. 1, Left the incoming current may consist of outgoing electrons while the outgoing current may be composed of outgoing positrons. This current brakes the rotation of the star through the torque created by the Lorentz force density $\mathbf{j} \times \mathbf{B}/c$ in the stellar atmosphere. Stellar angular momentum is then transported by the electric current (which twists the magnetic field) in the wind, and dumped somewhere far out in the wind. Where, is the big question.

The main problem with the pulsar wind is that apparently the value of σ in (1) changes from $\sigma \gg 1$ near the star to $\sigma \ll 1$ somewhere in the wind. The first value follows from the assumption that the radio pulsar is a strongly magnetized neutron star with surface fields $B_\star \sim 10^9$–10^{13} G whereas observations of the Crab nebula demonstrate that $\sigma \sim 2 \cdot 10^{-3}$ in the un-shocked wind (Fig. 1, Right). To solve this problem one needs to know where and how the electric current closes in the wind so as to dump the Poynting flux in the form of kinetic energy of the wind.

Our approach to the pulsar wind problem starts with the 1D, non-relativistic, stellar wind description in terms of an electric circuit in Sect. 2.1. We then include 2D effects in Sect. 2.2, consider the aligned magnetic rotator in vacuo in Sect. 2.3, discuss numerical results for the electric circuit in the aligned rotator in Sect. 2.4, consider the importance of current starvation in Sect. 2.5, consider the multiple effects of obliquity on the electric circuits in Sect. 2.6, critically review the nature of current sheets in Sect. 2.7, mention the effect of a 3D instability in Sect. 2.8, and conclude the first of the three parts of this review by summarizing our findings about the pulsar electric current circuit in Sect. 2.9.

2.1 Stellar Winds: 1D MHD, Partial Current Closure

Already in the early days of stellar MHD, Schatzman (1959) and Mestel (1968) realized that magnetic fields anchored in a star and extending into its atmosphere force an ionized stellar wind to co-rotate with the star to distances much larger than the photosphere. As a result the specific angular momentum carried off by a magnetized wind is much larger than its specific *kinetic* angular momentum at the stellar surface because of the increased lever arm provided by the magnetic field. Our starting point is the 1D MHD description of a magnetized stellar wind by Weber and Davis (1967). Their aim is to find a steady-state smooth wind. They assume the validity of ideal (i.e. non-resistive) MHD and consider an axially symmetric magnetic structure of a special kind—a so-called split magnetic monopole—which is obtained by reversing the magnetic field of a magnetic monopole in one half-sphere. This structure rotates around the axis of symmetry (*aligned* rotator), and it is assumed that the poloidal structure is not changed when rotation sets in. Finally, they consider the outflow just above the equatorial plane. They define an *Alfvén* radius r_A in the equatorial plane where the ram pressure of the radial flow part equals the radial magnetic tension. More generally, the Alfvén radius is determined by the *poloidal* components (Mestel 1999, Chap. 7):

$$\rho(r_A)v_{pol}^2(r_A) = \frac{B_{pol}^2(r_A)}{4\pi}, \tag{6}$$

or equivalently, where the poloidal flow speed equals the poloidal Alfvén speed (i.e. an Alfvénic Mach number unity). They find that the angular momentum loss per unit mass equals

$$r v_\phi - \frac{r B_r B_\phi}{4\pi\rho v_r} = \Omega_\star r_A^2. \tag{7}$$

Here r, ϕ are, formally, spherical radius, and azimuth respectively, but note that their derivation is valid just above the equatorial plane ($\theta = \pi/2$) so that r really is the cylindrical distance. Again the mass density ρ and the other quantities are measured in the observer (lab) frame. Their main finding then is that the total torque of a magnetized wind is equal to producing effective co-rotation of the wind out to the Alfvén radius. Nice and simple though this result is, it is also deceptive in that the current does not close at the Alfvén surface. Actually, most of the current goes off to infinity. The authors find that the distribution of angular momentum over electromagnetic and kinetic parts with distance follows Fig. 2, Left which shows that only a part of the Poynting torque is converted into a kinetic torque, and that asymptotically at large distance the Poynting contribution becomes constant and dominates. For the current closure in the wind then (which necessarily requires more than one dimension to describe) we can conclude that there is only *partial current closure*, and that most of the current goes out into the wind along the field lines in a force-free manner, i.e. asymptotically the remaining current flows along the magnetic field lines (Fig. 3, Right). Indeed, Weber and Davis (1967) find that the current within a magnetic surface satisfies

$$I(r) = \frac{c}{2} B_r r^2 \frac{v_\phi/r - \Omega_\star}{v_r}, \tag{8}$$

and becomes constant at large distances. Therefore, the result of this study—which turns out to be applicable to the wind from a *'slow magnetic rotator', here defined as emitting a wind mainly driven by thermal gas pressure*—is that at large distances the magnetic angular momentum and the Poynting energy flux dominate, so that $\sigma_\infty > 1$. In the next sections, we will investigate how the introduction of more realistic conditions and the transition to the pulsar wind open up the possibility of a small magnetization at infinity.

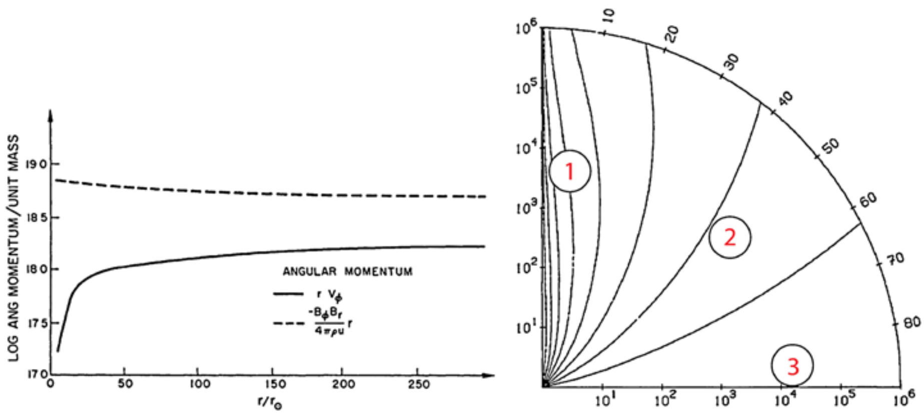

Fig. 2 *Left*: Partition of wind specific angular momentum over kinetic and magnetic contributions with distance, expressed in solar radii, for the Sun in 1D approximation; *dashed line* is magnetic torque and *drawn line* kinetic angular momentum. The Alfvén radius is between 30–50 solar radii. It follows that the torque remains ultimately electromagnetic, and that only a smaller part of the current closes at a finite distance. From Weber and Davis (1967), ©AAS. Reproduced with permission. *Right*: Focussing of a stellar wind in 2D approximation. As a result, there is subsequent complete conversion of Poynting into kinetic angular momentum. The computation is for an aligned rotator, an initially split monopole magnetic field, and an ideal MHD plasma. The bending back of the field lines towards the axis is not real and due to the logarithmic plotting of distance of a field line as a function of polar angle. Distance is in Alfvén radii. Credit: Sakurai (1985), reproduced with permission ©ESO

2.2 Stellar Winds: 2D, Complete Current Closure

The next major step in our understanding of a magnetized wind was taken by Sakurai (1985) who investigated the angular dependence of a steady, axially symmetric, stellar wind of an aligned rotator, again under the assumption of smooth, ideal MHD, and for an initially split monopole. Now, it is not sufficient to solve the 1D equation of motion (the *Bernoulli equation*) but at the same time the equation describing force balance in the meridional plane (the *trans-field equation*) is required. An important result of his study is that the entire Poynting flux is converted into kinetic energy. This result can be understood from the requirement of force-balance across the poloidal field. Figure 3, Right demonstrates the focussing of a magnetic wind towards the rotation axis. Three regimes can be distinguished in the wind solution:

1. In the polar region, a dense jet exists where gas pressure $-\nabla p$ is balanced by the contributions from the Lorentz force $-\nabla \frac{B_\phi^2}{8\pi}$ and $\frac{(\mathbf{B}_\phi \cdot \nabla)\mathbf{B}_\phi}{4\pi}$;
2. At intermediate latitudes, the magnetic field is force-free $\mathbf{j} \times \mathbf{B} = 0$, i.e. the magnetic pressure $-\nabla \frac{B_\phi^2}{8\pi}$ and the hoop stress $\frac{(\mathbf{B}_\phi \cdot \nabla)\mathbf{B}_\phi}{4\pi}$ are in balance;
3. Finally, in equatorial regions, the magnetic pressure $-\nabla \frac{B_\phi^2}{8\pi}$ balances the inertial force $-\rho(\mathbf{v} \cdot \nabla)\mathbf{v}$.

The result of magnetic focussing away from the equatorial plane is an inertial current density, given by the MHD momentum equation:

$$\mathbf{j} = c\frac{\mathbf{f} \times \mathbf{B}}{B^2}, \tag{9}$$

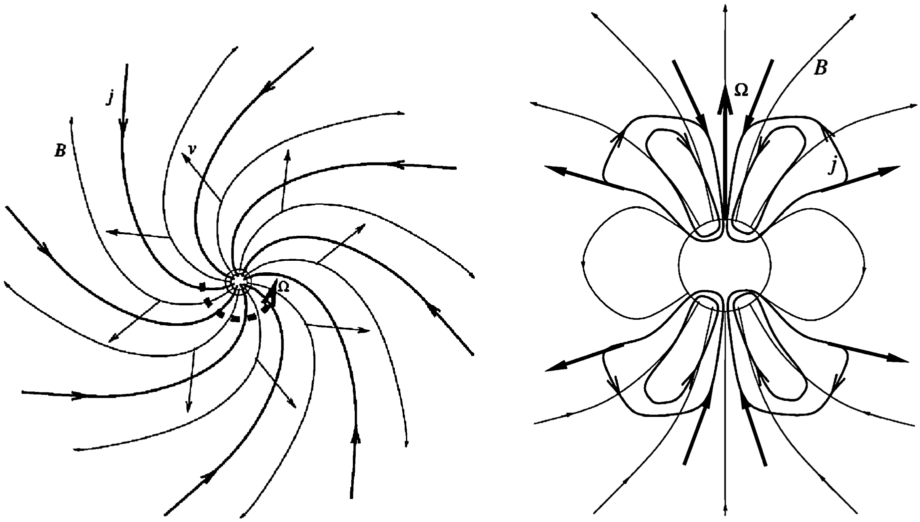

Fig. 3 *Left*: 2D sketch of the magnetic wind of an axially symmetric, aligned rotator just above the equatorial plane as seen by a poleward observer. B is the magnetic field, v the wind speed, j the electric current density, and Ω the rotation rate. *Right*: Sketch of the same structure as projected onto the meridional plane. Note that part—but not all—of the current is closing and thereby accelerating the wind close to the star. Adapted from Kuijpers (2001), ©Cambridge University Press, reprinted with permission

driven by the inertial force density

$$\mathbf{f} = -\rho(\mathbf{v} \cdot \nabla)\mathbf{v}. \tag{10}$$

Since the magnetic field has both a poloidal and a toroidal component the Lorentz force associated with the inertial current density not only focusses the wind in the poloidal plane toward the rotation axis but also accelerates the stellar wind radially outward, thereby converting magnetic into kinetic energy.

MHD Integrals for Axially Symmetric Cold Winds The axially symmetric case of an MHD plasma is especially illuminating because of the four integrals of motion allowed by the (2D) MHD equations (Mestel 1968). Here we consider cold gas, neglect gravity, but allow for relativistic motion (Lorentz factor Γ, axial distance r). In terms of quantities in the laboratory frame, these conserved quantities can be written as (Mestel 1999, Chap. 7):

$$\frac{B_\phi}{B_{pol}} = \frac{v_\phi - \Omega(\psi)r}{v_{pol}} \tag{11}$$

This is the *frozen-in field condition* which derives from the requirement that the gas exerts no stresses on the magnetic field. The parameter ψ labels the magnetic flux surfaces. Expressed in the co-rotating frame in which the field pattern is static, it amounts to requiring the gas to be moving along magnetic field lines.

$$\frac{B_{pol}}{4\pi\rho v_{pol}} = cF(\psi) \tag{12}$$

This equation is tantamount to a *constant mass flux* along a unit outgoing poloidal flux tube.

$$r\Gamma v_\phi - cF(\psi)rB_\phi = L(\psi) \tag{13}$$

If one multiplies this equation with the constant mass flux per unit poloidal flux tube one obtains the *conservation of total angular momentum flux* per unit poloidal flux tube as the matter moves out. This angular momentum is made up out of kinetic specific angular momentum and electromagnetic angular momentum.

$$\Gamma c^2 - cF(\psi) r\Omega(\psi) B_\phi = c^2 W(\psi) \tag{14}$$

Similarly, from this equation follows a generalization of Bernoulli's equation, the *conservation of total energy flux* per unit poloidal flux tube (for the cold gas) as the gas moves out. This total energy flux resides in kinetic (mass) energy flow and in Poynting flux.

2.3 Pulsar: Aligned Rotator; Vacuum Versus Plasma

Before we are in a position to study the degree of magnetic focussing in a pulsar wind we have to explain why an *aligned* magnetic rotator is relevant at all to the radio pulsar wind. Of course, a radio pulsar only exists if the magnetic rotator is *oblique*, or at least if deviations from axial symmetry exist. However, there are separate important electromagnetic effects which come in already for an aligned relativistic wind apart from the effects of obliquity, and we will try to disentangle these. Two basic consequences from Maxwell's theory require our attention:

– In vacuo, an axially symmetric magnetic rotator is—in a steady state—surrounded by an axially symmetric magnetic field. Since the stellar environs are assumed to be a vacuum no electric current flows. Any external electric fields are axially symmetric and therefore time-independent. Therefore, there is no displacement current. The absence of both material and displacement electric currents imply that the surrounding magnetic field is purely poloidal. Since the magnetic field is time-independent any electric field has to be poloidal as well. A *circulating* Poynting flux does exist in the toroidal direction but there is *no outgoing* Poynting flux. A fortiori then, there are no radiative electromagnetic losses.
– In the presence of an ionized wind, the situation is different. Now, poloidal electric currents do exist, both as a result of the unipolar induction by the rotating magnet, and because of the drag on the wind from the rotating field. As a result, the external magnetic field has a toroidal component. Also, because of the ideal MHD condition (2), an electric field appears in the lab frame which has a poloidal component. These electric and magnetic field components imply an outgoing Poynting flux (as in a common stellar wind). The magnetic field and the gas motion are, however, again time-independent, and as a result the electric field is time-independent as well. Therefore, there is no displacement current, and again no radiative losses (such as will be the case for an oblique rotator).

Thus, by confining ourselves to the aligned rotator first, we postpone the discussion about the importance of displacement electric fields, and isolate the Poynting flux which appears in the ideal MHD approximation. Such a pulsar wind is the (relativistic) extension of the above magnetized stellar wind. The electric circuit is expected to close along the path of least resistance, which means across the magnetic field inside the star and along the magnetic field in the wind. However, an important difference with an ordinary star appears. The neutron star may not be able to provide sufficient gas to short out the electric field component in the wind along the magnetic field. This happens, when the local charge density is not everywhere equal to the GJ charge density (4). There and then parallel potential drops develop.

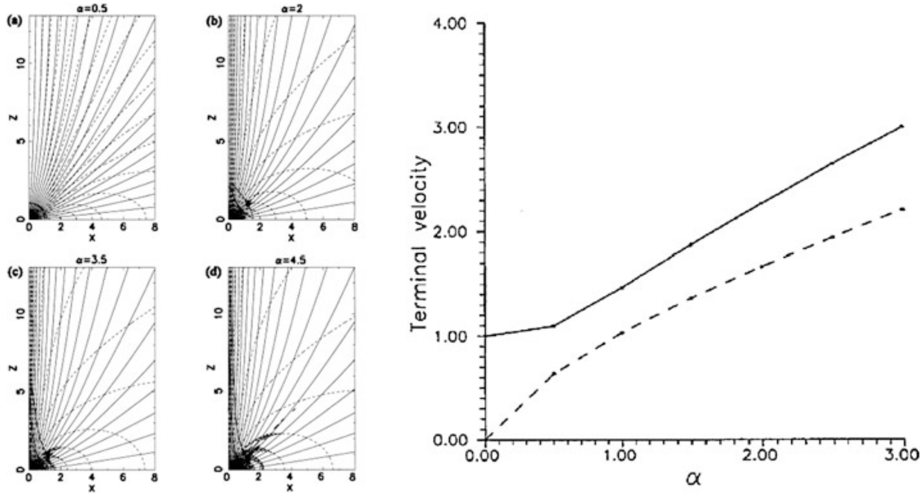

Fig. 4 *Left*: Sequence of shapes of the poloidal field lines (drawn) with increasing magnetic rotator parameter α from $\alpha = 0.5$ (solar-wind-type slow magnetic rotator) to $\alpha = 4.5$ (fast magnetic rotator). The initial non-rotating monopole magnetic field has a spherical Alfvén surface located at one Alfvén radius (r_A). Distances are in units of r_A with the base located at $x = 0.5$. *Dotted lines* indicate poloidal currents. *Thick lines* indicate Alfvén and fast critical surfaces. From Bogovalov and Tsinganos (1999), Fig. 2. *Right*: The terminal velocity as a function of α (*solid line*). For comparison, the corresponding terminal speed in Michel's minimum energy solution is also plotted (*dashed line*). From Bogovalov and Tsinganos (1999), Fig. 5

2.4 Aligned Rotator: Numerical Results on Collimation and Acceleration

Does the relativistic nature of the pulsar wind promote collimation and reduction of σ? The effects of fast rotation on collimation and acceleration of both non-relativistic and relativistic stellar winds have been investigated numerically by Bogovalov and Tsinganos (1999). They find both the collimation and the acceleration of the wind to increase with the *magnetic rotator parameter* (Bogovalov 1999)

$$\alpha \equiv \frac{\Omega r_A}{\Gamma_0 v_0}. \tag{15}$$

Here, Ω is the wind angular rotation rate (at most equal to the stellar rotation rate), v_0 the outflow speed at the photosphere, Γ_0 is the corresponding Lorentz factor, and r_A the Alfvén radius. The behavior of the non-relativistic outflow is shown in Fig. 4, Left. Since relativistic outflows such as in pulsars effectively have $\Gamma_0 \sim 100$ they belong to the domain of slow rotators with $\alpha \ll 1$, and collimation becomes ineffective (Fig. 4, Right; Fig. 5, Left).

The precise role of collimation for relativistic wind acceleration has been elucidated by Komissarov (2011). Consistent with his numerical results, he shows that *differential* collimation as sketched in Fig. 5, Right, is required to obtain high Lorentz factors. Such differential collimation is associated with increasing conversion efficiency of Poynting into kinetic energy flux. Tentatively, we conclude that aligned relativistic rotators lack sufficient differential collimation to establish substantial conversion of Poynting flux if the ideal MHD limit—including inertia—applies throughout the wind. Clearly, the relativistic wind motion does not help to reduce the wind magnetization.

The Effect of a Parallel Potential Gap Contopoulos (2005) shows that a constant potential gap at the basis of the open field lines allows the (otherwise ideal) wind to sub-rotate with