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# NEUTRON CROSS SECTIONS

*by*

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PERGAMON PRESS

LONDON · NEW YORK · PARIS

1957

PUBLISHED BY  
PERGAMON PRESS

*4 & 5 Fitzroy Square, London W.1*  
*122 East 55th Street, New York 22, N.Y.*  
*24 Rue des Écoles, Paris V<sup>e</sup>*

*Printed in Great Britain by Page Bros. (Norwich) Ltd., Norwich*

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## PREFACE

THE writing of this book is a direct result of experiences connected with the collection and evaluation of cross-section data during the past eight years at "Sigma Centre", Brookhaven National Laboratory. Here, experimental results received from laboratories throughout the world are carefully evaluated and compiled in the curves and tables of the large volume *Neutron Cross Sections*. The most recent version of the compilation, known as BNL 325, appeared 1 July 1955, and Supplement 1 to BNL 325 was published on 1 January 1957.

Hundreds of requests have come to Sigma Centre, concerning not only specific cross sections but the way in which to use them in practical applications or to assess their accuracy. Many people are now in a position where they must use cross-section data in their work but are handicapped because they have not had training in nuclear theory. Many of the cross-section requests can be answered by recourse to simple theoretical principles but of course this information, not being included in the compilation, is not easily available to those without nuclear training. Actually, it is often true that basic knowledge is needed even to *ask* a question in clear form about a cross section; again this information cannot be gained from the compilation itself.

Thus it became clear that there was a definite need for a clear and concise explanation of the principles of cross-section theory, measurement, and use that underly the compilation, in other words, a manual that would assist in the understanding and use of BNL 325. The compilation itself consists almost completely of cross sections at specific energies, shown in the form of curves or tables, with only brief explanatory texts. The present monograph, it is hoped, will be an answer to the need that has become evident in the course of compilation work. Its objective is to present the principles of cross-section measurement and use, as well as sufficient theory so that the general behaviour of cross sections is made understandable.

After a brief view of the most general properties of cross sections, Chapter 1, the book goes on to a discussion of the theory of cross sections and of nuclear structure. Here an attempt is made to present the basic facts of cross sections, without going into more details than are necessary for a clear and correct understanding. It is impossible to treat the theory of cross sections without some description of current views of nuclear structure, for the interactions

of neutrons with nuclei obviously are determined by the latter's structure. Those principles of nuclear structure that are particularly important to the understanding of cross-section behavior are therefore included in Chapter 2.

The specific techniques for measurement of cross sections and a consideration of the experimental results are then presented, beginning with fast neutrons in Chapter 3 and continuing with resonance neutrons in Chapter 4, resonances in fissionable materials in Chapter 5, and concluding with thermal neutrons in Chapter 6. In the discussions of experimental results in these chapters, actual cross sections are presented for illustrative purposes only, for the present monograph is not a compilation of data. It deals rather with the principles of cross-section measurement, behavior, and use, which hopefully will lose their pertinence only very slowly. The actual experimental results, which at times change rapidly, can be found in BNL 325 and its supplements and future editions. Frequent reference is made to BNL 325, however, for it is hoped that the present monograph will increase the usefulness of the compilation to a great extent.

Because this volume is intended for the many individuals who must use cross sections but do not have training in advanced nuclear theory, it was not felt desirable or necessary to include numerous references to original literature. For those with advanced training the compilation alone is probably sufficient, as it does contain references to the original sources of data. General references have been listed at the ends of the various chapters in the present volume that can be used for further reading and study.

I should like to express my gratitude to J. A. Harvey for his careful reading of the manuscript and valuable suggestions, as well as to G. Cox for many of the original drawings and to A. Marshall for preparation of the manuscript.

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## CHAPTER 1

# TYPES OF CROSS SECTIONS AND PRINCIPLES OF MEASUREMENT

THE subject of the interaction of neutrons with matter contains an astonishing diversity of phenomena. The basic reason for the great variety in their behaviour is that neutrons of an extremely wide energy range are able to interact effectively with nuclei. As the neutron carries no net electrical charge, there is no Coulomb repulsion to prevent its interaction with nuclei and it is thus able to cross the nuclear boundary even when moving at less than thermal velocity. Because of the lack of charge, the energy region over which neutrons cause nuclear interactions is much wider than it is for protons. Primarily because of this simple fact, a broad field of *neutron physics* exists—contrasted to which, of course, we cannot point to an equally wide field of “proton physics”. From nuclear reactors alone, neutron beams are available in sufficient intensity to make possible effective studies of interactions with matter from energies as low as  $10^{-4}$  eV to an upper limit at about  $10^7$  eV, an astonishingly large range in energy—a  $10^{11}$ -fold spread. Actually, including the most powerful accelerating machines presently available increases the energy spread only slightly, that is up to a few times  $10^9$  eV, the energy available with the Bevatron accelerator at the University of California.

Corresponding to the enormous range in energy over which neutrons exhibit appreciable interactions with matter, there is a great variety in the types of interactions that are observed. As the neutron energy increases throughout this energy range its wavelength decreases, from a value much greater than the distance between atoms in ordinary matter at the lowest energies, until at the upper limit the wavelength becomes much smaller than the size of the nucleus, eventually reaching a size about equal to the range of nuclear forces. Considering the  $10^{11}$  energy factor, it is not at all surprising that in this range the properties of neutrons in their interactions with matter can also vary greatly.

At the low energy or long wavelength limit of available intensity, the optical properties of neutrons predominate and we can demonstrate refraction and reflection of neutrons in ordinary matter. As the energy increases and the neutron wavelength decreases, the particle

characteristics of neutrons become more dominant until at the highest energies it seems most natural to think of neutrons as point projectiles colliding with the individual particles that make up the nuclei.

The measure of the interaction of neutrons with matter is the neutron *cross section*, and the great range of phenomena is reflected in the range of types of cross sections that can be measured. The collection of neutron cross sections shown as curves in the compilation\* BNL 325 illustrates this variety. For some cross sections there exist great differences from element to element or even from isotope to isotope, while for other cross sections there are striking regularities as one moves through the Periodic Table or from energy to energy within a single nuclide. The numerical magnitudes of the cross sections vary from values as large as millions of barns down to millionths of barns, that is, from *megabarns* to *microbarns*.

The matter of neutron cross sections is the very heart of neutron physics, for in one sense almost the whole of neutron physics is just the study of cross sections. Every neutron interaction can be described as a particular type of cross section, hence the measurement of any phenomenon with neutrons is automatically a cross-section measurement of some type. Fortunately, the great range of cross-section types exhibited in the compilation have a close connection with underlying theory, which, although it is not sufficiently exact to predict cross sections, nevertheless is of great value in correlating, interpolating between measured points, and even "guessing" many of them with fair accuracy.

#### SIMPLE DEFINITION OF CROSS SECTION AS NUCLEAR AREA

Cross sections are measured in units of area, the almost universally used "barn" (the basis for the cover design of BNL 325) being equal to  $10^{-24}$  cm<sup>2</sup>. Although in Chapter 2 we shall be concerned with the more exact quantum mechanical analysis of cross sections, at the present time a simple dimensional definition related to area is useful. The simple concept of cross section in terms of area is not only useful for its illustrative purposes, but is exact enough so that for practically all applications one can consider cross sections simply as target areas, with no error arising as a result. In this elementary sense, the cross section of a nucleus is just the target area presented by that nucleus to an approaching neutron. The target area or cross section

\* *Neutron Cross Sections*, D. J. Hughes and J. A. Harvey, July 1, 1955, Brookhaven National Laboratory Report BNL 325, and BNL 325-Supplement No. 1, D. J. Hughes and R. B. Schwartz, January 1, 1957, Superintendent of Documents, Government Printing Office, Washington 25, D. C. (\$3.50 and \$1.75).

is measured as area on a plane normal to the motion of the neutron, and can be considered simply as the area of the projection of the actual nucleus on the plane.

This picture of course must not be taken too literally because the cross section changes, and often very rapidly, with the velocity of the neutron, whereas it is obvious that the actual size of the nucleus is not dependent on the velocity of the incoming neutron. The cross section defined as the projected area of the nucleus is a measure of the probability that a neutron will hit that nucleus regardless of what happens after the collision. This type of cross section is called the *total cross section*, which is proportional to the total probability of interaction, of whatever kind, for the nucleus.

The total cross section can be subdivided into *partial cross sections*, each proportional to the probability of a particular event ensuing, of the various possible end-results following the collision of the neutron with the nucleus. For instance, the neutron may collide elastically and not be absorbed at all, it may be captured by the nucleus followed by emission of gamma radiation, it may emerge from the nucleus with its energy decreased, or it may even cause the nucleus to split, with the emission of perhaps a proton or an alpha particle or, for several heavy nuclei, by the more violent phenomenon that is fission. The partial cross sections are all measured in terms of area, as is the total cross section, and the sum of the partial cross sections of course must add to the total cross section itself. In this simple picture the target may be considered as divided into different sections, each corresponding to a particular type of reaction, and a neutron hitting a particular section resulting in the corresponding reaction. If the total cross section were always reasonably constant with energy, as it is at high energy, this picture of a subdivided nuclear area would seem reasonable enough. In the intermediate and low energy regions, however, the total target area itself must be considered to be changing rapidly with neutron energy, a property that is a bit difficult to reconcile with such a simple model.

From the model we have just considered, even if far too crude in some respects, the basic properties of neutrons in interacting with matter are nevertheless easily and correctly derived. Thus, as shown in Fig. 1-1, the number of interactions taking place per unit time when  $N'$  nuclei are bombarded by neutrons of density  $n$  per  $\text{cm}^3$  and velocity  $v$ , cm per sec, is given by:

$$\text{interactions per second} = N'nv\sigma_T, \quad (1-1)$$

assuming that  $N'$  is small enough so that the nuclei do not change the neutron density appreciably. Thus the interaction rate is proportional to the product of neutron density  $n$  and velocity  $v$ , which is the

*neutron flux*. Although strictly speaking  $nv$  should be called the flux density, wide acceptance of the shorter term has standardized its usage. Thus a unit cross section is one that gives one interaction per unit time when a unit neutron flux is incident on a single nucleus. It is to be noted that the interaction rate depends on the total number of nuclei,  $N'$ , not on the number per  $\text{cm}^3$ ,  $N$ , which we shall use in many other connections later.

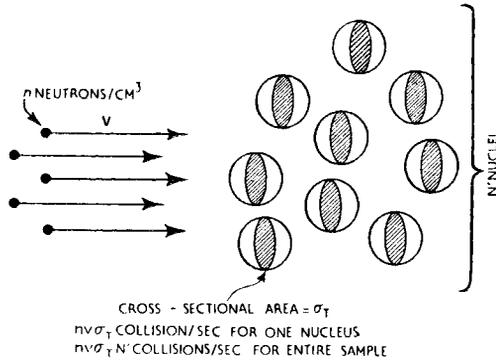


FIG. 1-1. The simple concept of neutron cross section as nuclear area. The number of neutrons in the volume  $v\sigma_T$  will hit a given nucleus per second. This number is  $nv\sigma_T$  for one nucleus or  $N'nv\sigma_T$  for all nuclei, regardless of the density of the sample or orientation, if  $N'$  is sufficiently small so that there is no self-shielding.

Although the practical utility of the concept of a cross section as nuclear area is obvious, its unreality is clearly shown by the variation in size of cross sections, enormous compared to the relatively constant true nuclear size. Only for high energy neutrons, for which the wavelength, which can be thought of as the neutron's size, is less than that of the nucleus, is the cross section about equal to the geometrical size of the nucleus itself. For a nucleus of radius  $R$  the area projected on a plane is  $\pi R^2$ , which for a heavy nucleus is about  $3 \times 10^{-24} \text{ cm}^2$  or 3 barns. Thus the total cross section for fast neutrons is typically of the order of a few barns, though the way in which this cross section is divided into various partial cross sections may differ greatly from element to element. The similarity of cross sections for fast neutrons from nuclide to nuclide and the very gentle change with energy is a direct result of the fact that for short wavelength the particle aspects of the neutrons are dominant. For a small, almost a point particle, hitting a strongly interacting sphere the total cross section should be approximately  $\pi R^2$ . That the total cross sections are not precisely this value is a matter of great importance to modern nuclear theory, as we shall see, and reveals that the

nucleus is a much more weakly interacting object than had been long assumed.

At lower neutron energy, where the neutron *wavelength* is larger than the nucleus, the situation becomes much more complicated, as the wave properties begin to emerge. When the neutron is much larger than the nucleus, it is quite reasonable to conclude that the chance of interaction is given not by the size of the nucleus but by the size of the neutron itself, which is about  $\pi\lambda^2$  rather than  $\pi R^2$  where  $\lambda$  is the neutron wavelength ( $\lambda = \lambda/2\pi$ ). For non-relativistic velocities, the neutron wavelength is given by the formula

$$\lambda = \frac{h}{mv} \quad , \quad (1-2)$$

where  $h$  is Planck's constant and  $mv$  is the momentum of the neutron. The wavelength is expressed in terms of the neutron energy,  $E(\text{eV})$ , as

$$\begin{aligned} \lambda &= h/\sqrt{2mE} \\ &= 2.86 \times 10^{-9}/\sqrt{E} \text{ cm} \\ &= 0.286/\sqrt{E} \text{ \AA} \text{ Angstrom units (\AA)} . \end{aligned} \quad (1-3)$$

The equations for the neutron wavelength are given in the simple non-relativistic form because the neutron energies with which we shall be concerned are sufficiently low so that the relativistic corrections are negligible. The wavelength is much larger than the nuclear radius for intermediate and low energy neutrons, for example  $\lambda$  is  $4.6 \times 10^{-10}$  cm for a one eV neutron, several hundred times larger than a typical nuclear radius.

In actuality, as a glance at BNL 325 will show, the cross section does not turn out to be simply the slowly varying  $\pi\lambda^2$ , but is vastly more complicated. In general, it is much smaller than  $\pi\lambda^2$ , but does reach values as large as  $4\pi\lambda^2$  under very special circumstances, as we shall see later. The reason for the low cross-section values and the rapid variations in energy is that the wave properties of the neutron necessitate a wave-mechanical treatment of the nuclear interaction and the results are not at all what one would expect from a classical view of an object of cross-sectional area  $\pi\lambda^2$  hitting one of area  $\pi R^2$ . In addition, the variation in cross section from nuclide to nuclide is extremely great, and at first sight may seem to be rather capricious. However, modern theory has gone a long way toward the understanding of neutron cross sections even in the energy region where the variations with energy and nuclide are abrupt and large.

#### CROSS-SECTION NOMENCLATURE

With such an enormous range of types of cross sections and numerical magnitudes it becomes a rather difficult task to distinguish

clearly the cross sections for specific interactions and to correlate measured results. Even the question of nomenclature of types of cross sections has given rise to increasing confusion as measurement techniques have become more subtle, and it is only recently that the nomenclature has begun to be well codified. Much of the difficulty in cross-section terminology arises because the terms sometimes refer to the phenomenon or manifestation by which the cross section is measured, such as capture cross section (detection of capture gamma rays), activation cross section (radioactivity of end product), etc., and at times to the theoretical picture that is used to describe the interactions, such as compound nucleus formation cross section, reaction cross section, etc. Whereas it is true that in many cases a cross-section type, based on a theoretical model, can be clearly identified with a specific cross section as it is manifested experimentally, in general a careful distinction must be made between a theoretical interpretation of what is going on and the phenomena as they are actually measured in experiments. For our purposes it is best to emphasize those cross-section types that are based on the principle of measurement primarily, rather than on theoretical constructs. The collection, correlation and evaluation of measured cross sections, as is done in BNL 325, for example, is in the frame of reference of experiments, with only a few general theoretical concepts as guides.

Even for cross-section terms that refer only to experimental conditions, however, there is a wide variety, reflecting the range of phenomena that are observed at various energies. Many of these refer to specific interactions that are observed in limited energy bands only, such as "phonon gain scattering" or "spin-dependent incoherent scattering" and these we shall consider later, in appropriate chapters. But in spite of the variety, there are some cross section terms of rather general meaning that are useful throughout the entire energy range, and it is desirable to define these before proceeding. Certainly the most universally measured cross section is the *total cross section*, which as we have seen represents all the interaction processes that may result from the collision of a neutron with an atom, and is thus the sum of all the *partial cross sections*. Simple as this definition may seem, its interpretation, especially at low energy, requires careful attention. At energies above one eV or so, the individual atoms do not affect each other and the cross section of a particular atom is the same whether it is isolated or associated with other atoms in a crystal. At lower energy, however, the neutron waves scattered by different atoms interfere and the observed scattering, hence cross section, can be a strong function of the physical state of the material, for example whether it is crystalline or liquid. In this case, as in all

others, the total cross section found in BNL 325 is obtained from Eq. (1-1), which means that it is averaged over the atoms as they exist in the sample and is not the cross section of an isolated atom. For exactness, we have referred in this paragraph to the "atom", although most of the cross section is contributed by the nucleus, particularly at energies above one eV. There is an appreciable magnetic interaction between the neutron and the atomic electrons, as well as a very small electrostatic interaction, but these are observable only at low energy where the atomic electrons have an appreciable form factor.

Of the various partial cross sections, the *scattering cross section* has a very definite meaning, being the cross section for all processes in which the only *particle* emitted in the interaction is a neutron. If this neutron should have energy identical with the incoming neutron energy the scattering is then called *elastic scattering*, and in the case that the energy changes it is *inelastic scattering*. The usual type of inelastic scattering is that in which energy is lost by the neutron and emitted as gamma radiation.

In general, neutron scattering is not isotropic, hence it is useful to specify a cross section as a function of the direction in which the neutron is emitted. Thus the *differential scattering cross section*, which of course in turn may be elastic or inelastic, gives the cross section for emission of the neutron at a particular angle. This differential scattering cross section is expressed as the scattering per unit solid angle and its integral over the total solid angle must equal the scattering cross section, elastic or inelastic. It is tempting in this connection to refer to the "total scattering cross section" but ambiguity is reduced if the adjective "total" is reserved for the total cross section alone.

For those interactions other than scattering, there are of course a number of corresponding partial cross sections, the *capture cross section*, for example, for those interactions in which the neutron is absorbed by the nucleus, and followed by emission of gamma rays.

The capture cross section is sometimes called the *absorption cross section*, but this term has been used with such a variety of meanings that it is wise to discourage its general use, and to define it on the spot if its use is desired. Another particular type of cross section, the *activation cross section*, refers to those interactions measured by the resulting radioactivity of the nucleus formed. The activation cross section *usually* is the same as the capture cross section, but activation can sometimes be a result of reactions other than neutron capture, such as the  $(n,p)$  and  $(n,\alpha)$  reactions, which are not considered capture cross sections. It is also true that neutron

capture can result in a stable nucleus, which strictly speaking, has zero activation cross section.

A very special cross section is that for *fission*, which occurs only for the heaviest elements, whose isotopes are called *fissionable* if fission occurs for slow neutrons. The cross-section term much used for fissionable isotopes is the absorption cross section, which has such a variety of meanings that its use has often led to ambiguities. The current meaning with regard to fissionable isotopes, such as  $U^{235}$ , is reasonably well established, however. For slow neutrons the total cross section of  $U^{235}$  is made up of scattering plus fission plus capture, where by capture we here of course mean the  $(n,\gamma)$  reaction only. In this case the fission cross section plus the capture cross section is usually called the absorption cross section, that is, the total cross section minus scattering. This "absorption cross section" is particularly valuable for comparison with theory and is found plotted for the fissionable isotopes in BNL 325, p. 298, for example.

A type of cross section that is analogous to the absorption cross section, as just described, is the *nonelastic cross section*. It has been defined in a number of ways but agreement seems now about reached that it signifies the total cross section minus the *elastic* scattering cross section. The fundamental *raison d'être* for it is that a particular technique, the "*sphere transmission*", described in Chapter 3, gives just this cross section. It often is identical with the inelastic scattering cross section, but in addition will include other processes, such as capture,  $(n,\alpha)$ ,  $(n,p)$ , fission, etc., if they are appreciable. In the energy region where nonelastic cross sections are usually measured, several MeV, they are approximately equal to inelastic scattering, for the other processes mentioned are in general much smaller.

As the phenomena observed with neutrons change with neutron energy, various types of cross sections, useful only in certain energy regions, appear. Thus, at a few eV, we talk about "free atom cross sections", whereas for neutrons of about  $10^{-4}$  to  $10^{-1}$  eV, where we are interested in the phenomena of neutron optics, such as reflection and diffraction, there is much discussion of the "coherent" and "incoherent" cross sections. There is usually no difficulty in terminology with these cross sections, which apply only in special circumstances, but the more general types of cross sections such as capture, total, nonelastic and inelastic, etc., are sufficiently general so that care should be used in applying them.

The nomenclature used for cross sections has until recently not existed in a universally accepted form and as a result a certain amount of confusion has arisen. However, the Nuclear Cross Sections Advisory Group of the U.S. Atomic Energy Commission

has now proposed some rules of nomenclature to unify cross-section definitions and notation. The recommendations apply to the notation with regard to experimentally determined quantities rather than the theoretical significance of the cross sections. The type of reaction is indicated by subscripts, such as  $\sigma_{np}$  for the  $(n,p)$  reaction and  $\sigma_{nT}$  for the total cross section. Of course, when neutrons only are considered as incident particles, the  $n$  can be omitted in the subscript, thus in the present monograph we shall use  $\sigma_T$  as the total cross section and  $\sigma_p$  as the  $(n,p)$  cross section. The experimental variables that apply to the particular cross section are indicated in the parentheses following the cross section symbol, thus  $E$  in the parentheses refers to the energy of the neutron, as  $\sigma_T(E)$ , and  $\theta$  in the parentheses denotes the differential cross section at the angle  $\theta$ , as  $\sigma_n(E; \theta)$  for differential elastic scattering.

Even though the notation is kept to a minimum in this system, it can still become rather complicated, especially when reactions emitting several products occur. An example is the  $(n,2n)$  reaction for an incident neutron of energy  $E$ , written in full as

$$\sigma_{n2n}(E; E', \theta) ,$$

which gives the differential cross section for emission of a neutron of energy  $E'$  at the angle  $\theta$ , with no specification for the other emitted neutron. However, for most of the reactions that we shall be considering the terminology retains its simplicity as we shall see in later chapters. Examples are  $\sigma_{nn}(E)$  for the cross section for elastic scattering, and  $\sigma_{nn}(E; \theta)$  for the differential elastic scattering at angle  $\theta$ , which we can simplify to  $\sigma_n(E)$  and  $\sigma_n(E; \theta)$ .

A cross section of some importance is the total minus the elastic cross section, which we have already defined as the *nonelastic cross section*; it is denoted by  $\sigma_{nX}(E)$  or  $\sigma_X(E)$ ,

$$\sigma_X = \sigma_T - \sigma_n . \quad (1-4)$$

The *inelastic scattering cross section*, for the process in which a neutron is emitted from a nucleus with an energy different from the incident energy, is denoted by  $\sigma_{nn'}(E)$ . If we wish to specify the energy of the emitted neutron  $E'$ , we then have the cross section  $\sigma_{nn'}(E; E')$  and further to specify the angle of emission we have the differential cross section  $\sigma_{nn'}(E; E', \theta)$ . Of course the first  $n$  in the subscript, which we have included for the moment, can be omitted if neutron cross sections only are under consideration. We shall not concern ourselves at the moment with the additional but straightforward complications of the inelastic scattering notation arising from the specification of the energies and angles of emission of the gamma rays produced in inelastic scattering. The real advantage of a universally adopted

system of notation is not in its capability of handling complex, but infrequent, situations but in attaining consistency for the frequently measured cross-section types.

### GENERAL PRINCIPLES OF CROSS-SECTION MEASUREMENT

There is a close correspondence between the methods by which cross sections are measured and their terminology because cross sections are classified in a way based closely on the methods of measurement. Perhaps when cross-section and nuclear theories are

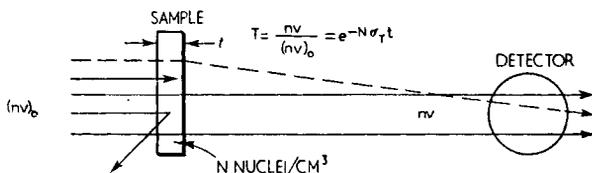


FIG. 1-2. The principle of the transmission measurement of the total cross section,  $\sigma_T$ . As the equation shown is based on the assumption that every interaction in the sample prevents a neutron from reaching the detector, correction must be made for the small angle scattering, shown by the dotted line, for which the neutron is still detected.

completely developed, cross sections may be described and discussed more in terms of their theoretical implication, that is, in terms of what is going on in the nucleus itself, rather than the experimental methods. However, at the present time, when the cross-section field is still somewhat in the empirical stage, the methods of measurement are still the important criteria for distinguishing types of cross sections. At present, we shall consider briefly the general principles of measurement techniques that apply throughout the entire energy range, reserving those specific to limited energies for later discussion.

By far the simplest type of cross section to measure, and furthermore, the one that can be measured most accurately, is the total cross section. In order to illustrate the principle of the measurement, let us for simplicity assume that we have a monoenergetic neutron source and also that the neutrons from the source are moving in a parallel incident beam as shown in Fig. 1-2. In this case the total cross section measurement is made simply by determining the *transmission* of the sample,  $T$ , that is, the ratio of the counting rate of the detector with the sample in the beam to that with no sample in place. The total cross section then follows from the relationship

$$T = \exp(-N\sigma_T t) , \quad (1-5)$$

where  $N$  is the number of atoms per cm<sup>3</sup> and  $t$  is the sample thickness.

A useful arithmetical time-saver is accomplished by writing Avogadro's number as 0.603 (omitting the  $10^{-24}$ ) and the cross section in barns, thus

$$N\sigma_T = 0.603 \frac{\rho}{A} \sigma_T, \quad (1-6)$$

with  $\rho$  the density of the sample and  $A$  the atomic weight. For iron, for example, and 1 eV neutrons,

$$\begin{aligned} N\sigma_T &= 0.603 \times 7.8 \times 11.6/55.9, \\ &= 0.98, \end{aligned}$$

and the transmission of a 1 cm thickness would be  $e^{-0.98}$  or 0.38.

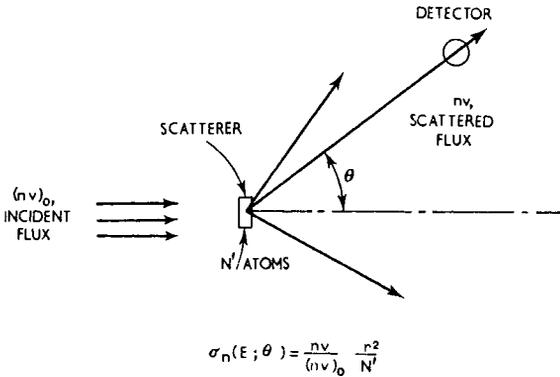


FIG. 1-3. The principle of the measurement of the differential scattering by detection of the scattered neutrons. The equation shown is based on the assumption that the sample is thin, hence does not attenuate the incident beam appreciably.

It is easy to see that any interaction in the sample that removes neutrons from the beam in any way, whether by scattering or a particular reaction, contributes to the total cross section measured in this way. We are not concerned at the moment with minor difficulties, such as correction for the fact that, with a finite size detector, the neutrons that are scattered by only a small angle will still hit the detector and hence their effect will be missed in the measurement. This, and other difficulties, can be avoided and the total cross section measured with extremely high accuracy, primarily because no absolute calibration of the detector is necessary, only the ratio of counting rates,  $T$ , entering the equation.

In principle, the elastic scattering cross section can be measured by moving the detector out of the direct beam, Fig. 1-3, so that it measures only the scattered neutrons at a particular angle. In this way the differential cross section is measured but, in contrast to the

total cross section, the absolute efficiency of the detector must usually be known. Furthermore, it must respond only to elastically scattered neutrons. The cross section cannot be obtained simply from a ratio of counting rates with the sample in place to an open beam because of the difference in shape and intensity of the direct and scattered neutron distributions. It is necessary to know  $\Delta\omega$ , the solid angle subtended by the detector at the scattering sample, in order to evaluate the scattered flux, hence the differential scattering cross section, in absolute terms. Thus the neutron flux measured at the angle  $\theta$ , relative to the incident flux, gives the differential elastic scattering cross section  $\sigma_n(E; \theta)$ :

$$\sigma_n(E; \theta) = \frac{nv}{(nv)_0} \frac{r^2}{N'} \quad , \quad (1-7)$$

where  $r$  is the scatterer-detector distance and  $N'$  the total number of atoms in the scatterer.

In a differential scattering measurement  $t$  must be sufficiently small so that there is little chance for neutrons to scatter more than once in the sample (i.e., negligible *multiple scattering*). Comparison of Eq. (1-7) for scattering with Eq. (1-5) shows that Eq. (1-7) contains the assumption that  $t$  is so small that there is negligible decrease in beam intensity in the sample ( $T$  near unity). Only a small fraction of the beam is scattered, and only  $\Delta\omega/4\pi$  of this fraction (if isotropic scattering) enters the detector, hence the lack of intensity in scattering measurements is easily appreciated. The elastic scattering cross section  $\sigma_n$  is determined from the integral over  $4\pi$  solid angle of the differential cross section, which, for isotropic scattering, is given simply as  $4\pi\sigma_n(E; \theta)$ .

Because of the complications mentioned, scattering cross sections in general are much more poorly known than total cross sections. We shall not discuss the methods used for the distinction of inelastic from elastic scattering at present for they are specific to fast neutrons primarily and will be taken up in detail in Chapter 3.

Even more difficult types of experiments are involved in the measurement of other partial cross sections such as capture,  $(n,p)$ ,  $(n,\alpha)$ ,  $(n,2n)$ , fission, etc. In the case of neutron capture it is necessary to detect the capture process by some specific property, the most direct being the emission of radiation. Determination of an absolute cross section, however, involving Eq. (1-1), means that the reaction rate must be obtained in absolute terms. The absolute rate is difficult to determine because the gamma rays emitted at neutron capture are usually unknown with regard both to energy and number per capture. Similar complications arise in connection with the other reactions listed, although for some, primarily fission, the absolute

efficiency of a detector is much easier to calibrate than in the case of gamma radiation. For all the reactions whose cross sections are measured in terms of reaction rates, however, it is essential that the absolute strength of the incident flux be known, a determination that is not at all simple. Without going further into the actual experimental details at the present time, this brief description should suffice to show why total cross sections can be measured simply and directly, at times with accuracies of the order of a few tenths of a per cent, while differential cross sections are much more difficult and the reaction cross sections more difficult still. In later chapters we shall consider the techniques and results of partial cross section measurements by a variety of methods useful for specific energy ranges only.

#### SURVEY OF EXPERIMENTAL RESULTS

Before proceeding to the more detailed discussions of cross section techniques and results, it seems wise to make a rapid survey of existing cross-section data as they are exhibited in the large compilation, BNL 325. We can thus gain a bird's-eye view of the types of cross sections, their variation with energy and with element, the accuracy with which they can be measured, and the extent to which information is available for the various cross-section categories.

At extremely low energies neutrons move so slowly that the manner in which they are scattered is affected greatly by the *thermal motion* of the atoms in the sample under investigation. Because of the important effect of the thermal agitation on scattering, the cross section for very slow neutrons varies greatly with the temperature of the sample. A striking example of this behaviour is afforded by the cross section of beryllium, shown in Fig. 1-4, which is a reproduction of the low energy curve for beryllium as given in BNL 325. The rapid variation of the total cross section with temperature is illustrated by the fact that the cross section at 440° K is 15 times greater than the cross section at 100° K. Because of the low capture cross section of beryllium, the total cross section is mainly scattering, which at very low energy is a type of inelastic scattering in which the neutron picks up energy from the lattice vibrations. In this type of scattering the neutron moves into the crystal very slowly, is then hit by a moving nucleus, and is knocked out of the crystal with much higher energy than its incident energy. A characteristic of lattice vibration scattering is that it increases with decreasing neutron energy, simply because the chance of a neutron's being hit by a moving nucleus is proportional to the time the neutron is in the scattering sample.

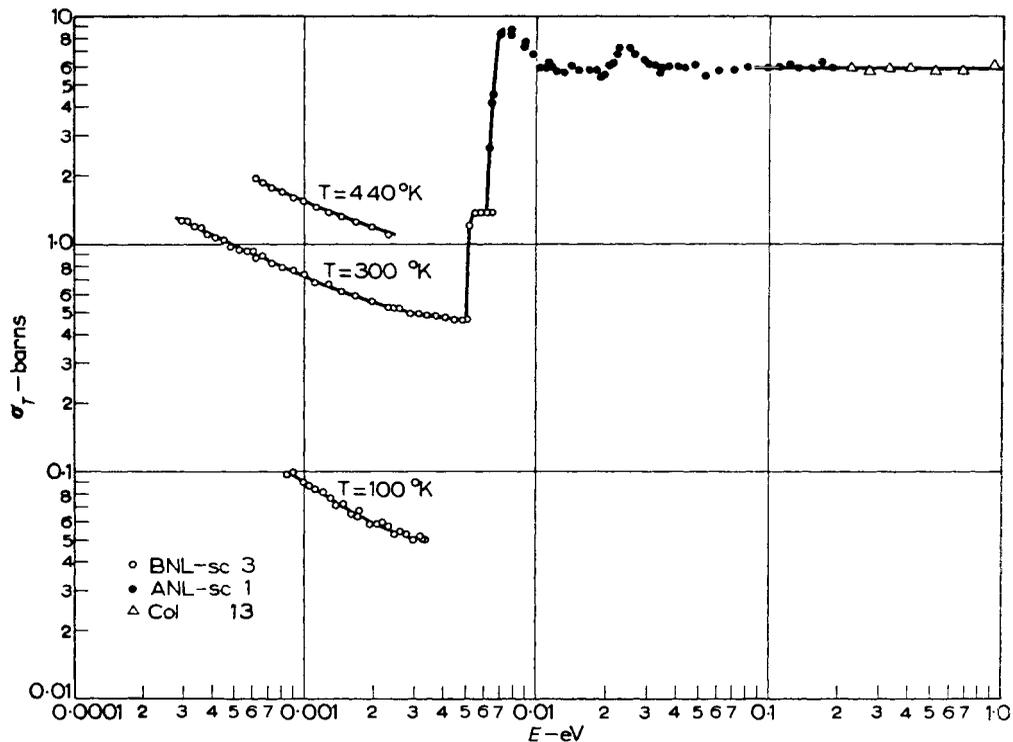


FIG. 1-4. The total cross section of beryllium in the low energy region, based on BNL 325; the references given by the code are listed at the end of the present volume. The rapid change of cross section with sample temperature is evidence for lattice vibration, or thermal inelastic, scattering.