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Thomas Meurer

Control of Higher–Dimensional PDEs

Flatness and Backstepping Designs

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Für Barbara

Preface

The analysis of distributed–parameter systems governed by partial differential equations (PDEs) has in recent years gained increasing attention and importance. This is on the one hand motivated by the need to continuously advance the technological state–of–the–art, which requires the detailed resolution of the temporal and the spatial dynamics of the involved devices. On the other hand, new application areas emerge, which essentially rely on the exploitation of the spatial–temporal system evolution. Examples include smart, flapping, and flexible structures in aerospace applications, fusion reactors and electrochemical devices for energy generation and storage, quantum systems and quantum computing, or interconnected systems in mobile actuator and sensor networks and traffic congestion. The dynamic operation of these multi–physics systems inherently requires sophisticated control and observer strategies that explicitly address the spatial–temporal system dynamics. For this, infinite–dimensional system and control theory has been developed and continuously refined to provide a unifying mathematical framework to address the arising analysis and design tasks.

Based on the steady progress in nonlinear control and its success in applications novel and promising lines of development have been identified by their generalization to distributed–parameter systems. This, for instance, comprises differential flatness for inversion–based trajectory planning and feedforward control and Lyapunov–based techniques for (robust) feedback stabilization and observer design such as backstepping, dissipativity, or passivity concepts. Moreover, tracking controllers can be deduced from the suitable composition of these approaches. While many significant contributions have already been achieved for the control design of mainly linear systems the analysis and control of distributed–parameter systems involving nonlinearities and higher–dimensional spatial domains poses challenges for both methodic and applied research.

This research monograph aims at addressing some of these challenges by presenting recent developments for the control of systems governed by PDEs. Herein, the main focus is on systems with higher–dimensional spatial domain by suitably developing systematic model–based design approaches to realize trajectory planning and feedforward control, feedback stabilization and observer design, and trajectory tracking. Techniques including flatness and backstepping build the core of the treatise and provide the starting point of the presented methodic extensions and generalizations. Moreover, semi–numerical approaches are deduced by integrating suitable methods of approximation into the design systematics to enlarge the domain of applicability. The theoretic analysis is combined with both simulation examples and state–of–the–art experimental results, which enables to bridge the gap between mathematical theory and control engineering practice in the rapidly evolving PDE control area.

The presentation is split into five parts starting with an introduction and survey of PDE control in Part I. Mathematical modeling of heat transfer problems, multi–agent networks, and flexible structures is considered in Part II. The derived model equations serve as the basis for flatness–based trajectory planning and feedforward control in Part III for PDE systems with both in–domain and boundary control. Part IV addresses feedback stabilization and observer design by generalizing backstepping techniques and considering their combination with trajectory planning and feedforward control towards the realization of tracking controllers for PDE systems. Simulation and experimental results are provided in the individual chapters to support the methodic developments and to illustrate their application. Selected mathematical tools and results from complex analysis, entire function theory, and functional analysis are summarized in Part V. Thus, the book is adequate as an advanced research monograph for graduate students in applied mathematics and control theory or as a reference to recent developments for researchers and control engineers interested in the analysis and the control of systems governed by PDEs.

This book would not have been possible without help, support, and suggestions of colleagues and family. In particular I am indebted to Andreas Kugi for his support and suggestions, many fruitful discussions, and his academic and personal advice. I want to acknowledge my students and co–workers in Vienna, especially the members of my work group Johannes Schröck, Lukas Jadachowski, Tilman Utz, and Marc Oliver Wagner. Moreover, I am particularly grateful to Miroslav Krstic for his suggestions and advice. During the work on PDEs I have enjoyed the interaction with Ansgar Jüngel, Pierre Rouchon, Birgit Schörkhuber, Kurt Schlacher, Andrey Smyshlyaev, Rafael Vazquez, Michael Zeitz. I am also thankful to my Viennese colleagues at the CDS group for collectively creating an open and friendly environment. In addition, the financial support of the German Research Council (DFG) is gratefully acknowledged. Last but not least I would like to thank my parents for their continuous support and Barbara for her encouragement and patience in the course of creating this book.

Vienna, January 2012 Thomas Meurer

Contents

List of Symbols and Abbreviations

The following list only contains symbols that are used consecutively throughout the text neglecting only locally introduced symbols.

Coordinates and Units

$\imath$	Imaginary unit $\imath^2 = -1$
$\mathtt{n}_j$	Node in communication graph with integer or multi–index j
t	Time coordinate
z^i	Spatial coordinate
z	Coordinate r–tuple $z = (z^1, z^2, \ldots, z^r)$
$z_{(i\vert)}$	Coordinate $(r-1)$–tuple $z_{(i\vert)} = (z^1, z^2, \ldots, z^{i-1}, z^{i+1}, \ldots, z^r)$
$z_{(i\vert L_i, j\vert L_j)}$	Coordinate r–tuple z with $z^i = L_i$ and $z^j = L_j$
$z_{(i\vert L_i)}$	Coordinate r–tuple z with $z^i = L_i$
$z_{(i\vert\zeta)}$	Coordinate r–tuple z with $z^i = \zeta$
s	Laplace variable

Variables

$\xi^{*,k}(z,t)$	k–th component of desired basic output vector with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\hat{\xi}(z,s)$	Laplace transform of $\xi(z,t)$
$\boldsymbol{\xi}(z,t)$	Basic output (vector) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{\xi}^*(z,t)$	Desired basic output (vector) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\hat{\boldsymbol{\xi}}(z,s)$	Laplace transform of $\boldsymbol{\xi}(z,t)$
$\xi^k(z,t)$	k–th component of basic output vector with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\xi(z,t)$	Basic output (scalar) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\xi^*(z,t)$	Desired basic output (scalar) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\tilde{w}(z,t)$	State of target system for state–observer design with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$w(z,t)$	State of target system for state–feedback design with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$u^k(\mathtt{n}_j,t)$	k–th component of input vector with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$u^k(z,t)$	k–th component of input vector with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$u_{\partial\Omega}$	Scalar boundary input (where distinguished from in–domain control u_Ω)

$u^{*,k}(\mathtt{n}_j,t)$	k–th component of feedforward control with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$u^*(\mathtt{n}_j,t)$	Feedforward control (scalar) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$u^{*,k}(z,t)$	k–th component of feedforward control with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$u^*(z,t)$	Feedforward control (scalar) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
u_Ω	Scalar in–domain input (where distinguished from boundary control $u_{\partial\Omega}$)
$\hat{u}(z,s)$	Laplace transform of $u(z,t)$
$u(\mathtt{n}_j,t)$	Input (scalar) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$u(z,t)$	Input (scalar) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{u}(\mathtt{n}_j,t)$	Input (vector) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$\boldsymbol{u}(z,t)$	Input (vector) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{u}_{\partial\Omega}$	Boundary input vector (where distinguished from in–domain control $\boldsymbol{u}_\Omega$)
$\boldsymbol{u}^*(\mathtt{n}_j,t)$	Feedforward control (vector) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$\boldsymbol{u}^*(z,t)$	Feedforward control (vector) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{u}_\Omega$	In–domain input vector (where distinguished from boundary control $\boldsymbol{u}_{\partial\Omega}$)
$\hat{\boldsymbol{u}}(z,s)$	Laplace transform of $\boldsymbol{u}(z,t)$
$\hat{x}(z,t)$	Observer state with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\tilde{x}(z,t)$	Observer error state with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{x}(\mathtt{n}_j,t)$	State (vector) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$x^k(z,t)$	k–th component of state vector with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$x^*(\mathtt{n}_j,t)$	Desired state (scalar) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$x^*(z,t)$	Desired state (scalar) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\hat{x}(z,s)$	Laplace transform of $x(z,t)$
$x^k(\mathtt{n}_j,t)$	k–th component of state vector with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$x(\mathtt{n}_j,t)$	State (scalar) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$x(z,t)$	State (scalar) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{x}(z,t)$	State (vector) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{x}^*(\mathtt{n}_j,t)$	Desired state (vector) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$\boldsymbol{x}^*(z,t)$	Desired state (vector) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\hat{\boldsymbol{x}}(z,s)$	Laplace transform of $\boldsymbol{x}(z,t)$
$y^k(\mathtt{n}_j,t)$	k–th component of output vector with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$y^k(z,t)$	k–th component of output vector with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$y^{k,*}(\mathtt{n}_j,t)$	k–th component of desired output vector with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$y^*(\mathtt{n}_j,t)$	Desired output (scalar) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$y^{k,*}(z,t)$	k–th component of desired output vector with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$y^*(z,t)$	Desired output (scalar) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\hat{y}(z,s)$	Laplace transform of $y(z,t)$
$y(\mathtt{n}_j,t)$	Output (scalar) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$y(z,t)$	Output (scalar) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{y}(\mathtt{n}_j,t)$	Output (vector) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$

$\boldsymbol{y}(z,t)$	Output (vector) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\boldsymbol{y}^*(\mathtt{n}_j,t)$	Desired output (vector) with $(\mathtt{n}_j,t) \in \mathtt{N} \times \mathbb{R}^+_{t_0}$
$\boldsymbol{y}^*(z,t)$	Desired output (vector) with $(z,t) \in \Omega \times \mathbb{R}^+_{t_0}$
$\hat{\boldsymbol{y}}(z,s)$	Laplace transform of $\boldsymbol{y}(z,t)$

Dimensions

m	Dimension of $\boldsymbol{u}$
n	Dimension of $\boldsymbol{x}$
p	Dimension of $\boldsymbol{y}$

Bases, Vectors, Matrices, and Operators

$\mathfrak{A}^*$	Adjoint of $\mathfrak{A}$
$r^a,\ r^g$	Algebraic and geometric multiplicity of eigenvalue λ
θ_k	Constant $\theta_k = 1 + r^a_k - r^g_k$
Δ	Laplace operator $\Delta = \nabla \cdot \nabla$
$\mathcal{D}(\mathfrak{A})$	Domain of $\mathfrak{A}$
ψ	Basis function or eigenfunction of $\mathfrak{A}^*$
$\boldsymbol{\psi}$	Base vector or eigenvector of $\mathfrak{A}^*$
ϕ	Basis function or eigenfunction of $\mathfrak{A}$
$\boldsymbol{\phi}$	Base vector or eigenvector of $\mathfrak{A}$
λ	Eigenvalue of $\mathfrak{A}$
∇	Nabla operator
$\mathfrak{A}$	System operator
$\mathfrak{B}$	Input operator
$\mathfrak{C}$	Output operator
$\mathfrak{K}$	Boundary input operator
$\mathfrak{I}$	Identity operator or identity matrix
$\mathfrak{o}$	Zero operator or zero matrix

Function Spaces and Classes

$\mathcal{L}(X,Y)$	Space of bounded linear operators from X to Y
$\mathcal{L}(X)$	Space of bounded linear operators from X to X
$C^n(\Omega)$	Class of n–times continuously differentiable functions from Ω to $\mathbb{C}$ or $\mathbb{R}$
$G_{D,\alpha}(\Lambda)$	Gevrey class of order α on domain Λ
$G_{D,\alpha,\beta}(\Lambda)$	Gevrey class of order α in the first argument and β in the second argument on domain Λ
$H^p(\Omega)$	Sobolev space of order p
ℓ^∞	Space of bounded sequences $(f_k)_{k\in\mathbb{N}^r}$ with $\sup_{k\in\mathbb{N}} \lvert f_k\rvert < \infty$
$L^\infty(\Omega)$	Class of Lebesgue measurable bounded functions from Ω to $\mathbb{C}$
ℓ^p	Space of sequences $(f_k)_{k\in\mathbb{N}^r}$ with $\sum_{k\in\mathbb{N}} \lvert f_k\rvert^p < \infty$
$L^p(\Omega)$	Class of Lebesgue measurable functions with $\int_\Omega \lvert f(t)\rvert^p \mathrm{d}t < \infty$
$L^p(\Omega;X)$	Class of Lebesgue measurable X–valued functions with $\int_\Omega \lvert f(t)\rvert^p \mathrm{d}t < \infty$

$\ell_k^2(0,\tau)$	Space of square summable sequences $(f_k(t))_{k\in\mathbb{N}^r}$ on the interval $t \in [0,\tau]$, i.e., $\forall t \in [0,\tau]:\ \sum_{k\in\mathbb{N}^r} \|f_k(t)\|^2 < \infty$
X_{-1}	see Lemma B.4
X_1	see Lemma B.3

Entire and Special Functions

γ	Convergence exponent (see Section B.2.2)
$\mathcal{N}(\eta)$	Counting function (see Section B.2.2)
g^f	Genus of entire function (see Section B.2.2)
ϱ	Order of entire function (see Section B.2.2)
τ	Type of entire function (see Section B.2.2)
σ	Heaviside function
$\sigma(t-t_0)$	Heaviside function with $\sigma(t-t_0)=0$ for $t<t_0$ and $\sigma(t-t_0)=1$ else
δ_{ij}, $\delta_{i,j}$	Kronecker delta function with $\delta_{ij}=\delta_{i,j}=1$ if $i=j$ and zero else
$\varrho^\epsilon(z)$	Indicator function satisfying $\varrho^\epsilon(z<0)=0$, $\varrho^\epsilon(z>\epsilon)=1$, and $\varrho^\epsilon(z)\in C^4([0,\epsilon])$ for $\epsilon>0$. If $\epsilon=0$, then $\varrho^0(z)=\sigma(z)$.
$\mathcal{G}$	Weierstrass primary factor (see Section B.2.2)
Π	Weierstrass canonical product (see Section B.2.2)
g^s	Genus of sequence (see Section B.2.2)

Integral Kernels

$k(z^i,\zeta,t)$	Kernel of backstepping transformation in state–feedback control design
$g(z^i,\zeta,t)$	Kernel of inverse backstepping transformation in state–feedback control design
$l(z^i,\zeta,t)$	Kernel of backstepping transformation in state–observer design
$m(z^i,\zeta,t)$	Kernel of inverse backstepping transformation in state–observer design

Sets

$\mathbb{C}$	Set of complex numbers
$\ker$	Kernel
$\mathbb{N}$	Set of natural numbers $\{0,1,2,\ldots\}$
$\mathbb{N}_1$	Set of natural numbers excluding 0
$\mathtt{N}$	Set of nodes (vertex set)
ran	Range
$\mathbb{R}$	Set of real numbers
$\mathbb{R}_{t_0}^+$	Set of real numbers $t\in\{t\in\mathbb{R}:\ t>t_0\}$
I_r	Index set $I_r=(1,2,\ldots,r)$
I_r^i	Index set $I_r^i=I_r\setminus\{i\}=(1,2,\ldots,i-1,i+1,\ldots,r)$
I_m	Index set $I_m=(1,2,\ldots,m)$
I_m^i	Index set $I_m^i=I_m\setminus\{i\}=(1,2,\ldots,i-1,i+1,\ldots,m)$
I_p	Index set $I_p=(1,2,\ldots,p)$
I_p^i	Index set $I_p^i=I_p\setminus\{i\}=(1,2,\ldots,i-1,i+1,\ldots,p)$
$\mathbb{Z}$	Set of integers $\{\ldots,-2,-1,0,1,2,\ldots\}$
$\mathbb{Z}_1$	Set of integers excluding 0

Algebra

$\#$	Number of elements in a set
$\bigtimes$	Cartesian product $\bigtimes_{j\in\{1,\dots,r\}}[0,L_j]=[0,L_1]\times\ldots\times[0,L_r]$
$\wedge$	Logical conjunction
$\vee$	Logical disjunction

Abbreviations

2DOF	Two–degrees–of–freedom
BVP	Boundary–value problem
DCRS	Diffusion–convection–reaction system
DPS	Distributed–parameter system
DRS	Diffusion–reaction system
IEOK	Integral equation with operator kernel
IVP	Initial–value problem
MFC	Macro–fiber composite
MIMO	Multiple inputs multiple outputs
ODE	Ordinary differential equation
PDE	Partial differential equation
PdE	Partial difference equation
SISO	Single input single output

Chapter 1
Introduction

Although exhibiting a long history and tradition, partial differential equations (PDEs) are just nowadays starting to evolve as the fundamental mathematical description of many technical processes. It can be thereby observed that the success story of PDEs entering new areas of applied research is related to the vast progress in information technology and (computational) mathematics, which provide access to an almost unlimited computational power and newly developed efficient algorithms. With this, the attention is focused on previously unthought problems such as high resolution climate simulation using increasingly finer spatial grids covering the earth's surface or the study of multi–phase compressible reactive flows in complex geometrical domains.

In general, the distributed–parameter description becomes an essential ingredient of the modeling and analysis process if the spatial or property–related distribution of the process variables can no longer be neglected. Some characteristic examples are summarized below:

- chemical or biochemical reactors [65] such as three-way catalysts for exhaust after–treatment in automotive applications, (reactive) distillation and adsorption processes [140], or activated sludge processes for wastewater treatment [75];
- thermal systems [6] or the reheating and cooling of metal slabs during the steel processing to achieve desired metallurgical changes [146];
- electrochemical systems such as fuel cells [141] and Li–ion or Li–polymer battery devices for energy production and storage [26, 58];
- smart materials and vibratory systems [88, 10, 109];
- flexible structures arising in aerospace and mechanical applications including novel adaptive or flapping wing structures [139], micro–mechanic bending cantilevers in atomic force microscopes [15], or deformable mirrors in adaptive optics [114];
- fluid dynamical systems [1, 14], mixing processes and coupled fluid–structure interactions;
- wave propagation in optical fibers [130] and traffic congestion [152, 61];
- energy production in fusion reactors [142, 3].

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However, the dynamic operation of these distributed–parameter systems (DPSs) essentially relies on the incorporation of suitable control strategies to influence the system dynamics and to enlarge the operating range. For this, it can be observed that in addition to the stabilization problem, the consideration of the trajectory tracking control problem, i.e. the design of a control such that the controlled variables of the distributed–parameter system follow prescribed desired reference trajectories, has gained increasing attraction. This is in particular due to the rising demands on product quality and efficiency, which require to turn away from the pure stabilization of an operating point towards the realization of specific start–up, transition, or tracking tasks. Selective academic and industrial applications, which illustrate and confirm the increasing interest in the study of tracking control problems for distributed–parameter systems can be found, e.g., in [108, 33, 100] and the references therein.

Nevertheless, although being of increasing practical interest, traditional control design approaches seem to provide only indirect and often unsatisfactory solutions to the tracking control problem. This is mainly due to the methodical focus on the feedback stabilization of a given distributed–parameter system without the consideration of the assignment and the realization of a suitable desired spatial–temporal dynamics.

1.1 Feedback Stabilization of PDE Systems

Model–based feedback control for distributed–parameter systems[1] can in general be classified in terms of early and late lumping. In the early lumping approach, the governing PDEs are reduced to a finite–dimensional description by making use of suitable approximation and model reduction techniques prior to the feedback control design. This includes the application of finite difference or finite element techniques [88, 109], modal or spectral approaches [56, 8, 88, 51, 54], balanced truncation and proper orthogonal decomposition [4, 11, 131], or Galerkin's method [111, 1] and its variants such as (approximate) inertial manifold techniques [49, 28, 27, 64]. Thereby, well–developed feedback control design methods can be applied originating from linear and nonlinear finite–dimensional control theory. However, depending on the order of approximation, the early lumping approach may lead to high–dimensional and complex feedback control structures, which do not fully exploit the physical structure of the system. In addition, the neglected dynamics may lead to a degradation of the control performance or even the destabilization of the closed–loop system due to the well–known control and observer spillover [7]. Furthermore, for nonlinear distributed–parameter systems the validity of the finite–dimensional approximation and hence the determined controller is usually restricted to a certain subset of the state space. As a result, situations may arise, where the

[1] In this section, only a brief overview of the major analysis as well as design approaches and techniques will be given, which are available for control problems governed by distributed–parameter systems. This can naturally include only a selection of methods and literature without any claim of being comprehensive.