INSTRUCTOR'S SOLUTIONS MANUAL

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with art by

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LINEAR ALGEBRA WITH APPLICATIONS

FOURTH EDITION

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Colby College

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Section 1.1

1.1.**1** $\begin{bmatrix} x+2y=1 \\ 2x+3y=1 \end{bmatrix} \begin{matrix} \\ -2\times\text{1st equation} \end{matrix} \rightarrow \begin{bmatrix} x+2y & =1 \\ -y & =-1 \end{bmatrix} \div(-1) \rightarrow$

$\begin{bmatrix} x+2y=1 \\ y=1 \end{bmatrix} \begin{matrix} -2\times\text{2nd equation} \\ \end{matrix} \rightarrow \begin{bmatrix} x & =-1 \\ y & =1 \end{bmatrix}$, so that $(x,y)=(-1,1)$.

1.1.**2** $\begin{bmatrix} 4x+3y & =2 \\ 7x+5y & =3 \end{bmatrix} \begin{matrix} \div 4 \\ \end{matrix} \rightarrow \begin{bmatrix} x+\frac{3}{4}y & =\frac{1}{2} \\ 7x+5y & =3 \end{bmatrix} \begin{matrix} \\ -7\times\text{1st equation} \end{matrix} \rightarrow$

$\begin{bmatrix} x+\frac{3}{4}y & =\frac{1}{2} \\ -\frac{1}{4}y & =-\frac{1}{2} \end{bmatrix} \begin{matrix} \\ \times(-4) \end{matrix} \rightarrow \begin{bmatrix} x+\frac{3}{4}y & =\frac{1}{2} \\ y & =2 \end{bmatrix} \begin{matrix} -\frac{3}{4}\times\text{2nd equation} \\ \end{matrix} \rightarrow \begin{bmatrix} x & =-1 \\ y & =\ \ 2 \end{bmatrix}$,

so that $(x,y)=(-1,2)$.

1.1.**3** $\begin{bmatrix} 2x+4y & =3 \\ 3x+6y & =2 \end{bmatrix} \begin{matrix} \div 2 \\ \end{matrix} \rightarrow \begin{bmatrix} x+2y & =\frac{3}{2} \\ 3x+6y & =2 \end{bmatrix} \begin{matrix} \\ -3\times\text{1st equation} \end{matrix} \rightarrow \begin{bmatrix} x+2y & =\ \ \frac{3}{2} \\ 0 & =-\frac{5}{2} \end{bmatrix}$.

So there is no solution.

1.1.**4** $\begin{bmatrix} 2x+4y & =2 \\ 3x+6y & =3 \end{bmatrix} \begin{matrix} \div 2 \\ \end{matrix} \rightarrow \begin{bmatrix} x+2y & =1 \\ 3x+6y & =3 \end{bmatrix} \begin{matrix} \\ -3\times\text{1st equation} \end{matrix} \rightarrow \begin{bmatrix} x+2y & =1 \\ 0 & =0 \end{bmatrix}$

This system has infinitely many solutions: if we choose $y=t$, an arbitrary real number, then the equation $x+2y=1$ gives us $x=1-2y=1-2t$. Therefore the general solution is $(x,y)=(1-2t,t)$, where t is an arbitrary real number.

1.1.**5** $\begin{bmatrix} 2x+3y & =0 \\ 4x+5y & =0 \end{bmatrix} \begin{matrix} \div 2 \\ \end{matrix} \rightarrow \begin{bmatrix} x+\frac{3}{2}y & =0 \\ 4x+5y & =0 \end{bmatrix} \begin{matrix} \\ -4\times\text{1st equation} \end{matrix} \rightarrow$

$\begin{bmatrix} x+\frac{3}{2}y & =0 \\ -y & =0 \end{bmatrix} \begin{matrix} \\ \div(-1) \end{matrix} \rightarrow \begin{bmatrix} x+\frac{3}{2}y & =0 \\ y & =0 \end{bmatrix} \begin{matrix} -\frac{3}{2}\times\text{2nd equation} \\ \end{matrix} \rightarrow \begin{bmatrix} x & =0 \\ y & =0 \end{bmatrix}$,

so that $(x,y)=(0,0)$.

1.1.**6** $\begin{bmatrix} x+2y+3z & =\ \ 8 \\ x+3y+3z & =10 \\ x+2y+4z & =\ \ 9 \end{bmatrix} \begin{matrix} \\ -I \\ -I \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z & =8 \\ y & =2 \\ z & =1 \end{bmatrix} \begin{matrix} -2(II) \\ \\ \\ \end{matrix} \rightarrow$

$\begin{bmatrix} x+3z & =4 \\ y & =2 \\ z & =1 \end{bmatrix} \begin{matrix} -3(III) \\ \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x=1 \\ y=2 \\ z=1 \end{bmatrix}$, so that $(x,y,z)=(1,2,1)$.

1.1.**7** $\begin{bmatrix} x+2y+3z & =1 \\ x+3y+4z & =3 \\ x+4y+5z & =4 \end{bmatrix} \begin{matrix} \\ -I \\ -I \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z & =1 \\ y+z & =2 \\ 2y+2z & =3 \end{bmatrix} \begin{matrix} -2(II) \\ \\ -2(II) \end{matrix} \rightarrow \begin{bmatrix} x+z & =-3 \\ y+z & =\ \ 2 \\ 0 & =-1 \end{bmatrix}$

This system has no solution.

1.1.8 $\begin{bmatrix} x+2y+3z & =0 \\ 4x+5y+6z & =0 \\ 7x+8y+10z & =0 \end{bmatrix} \begin{matrix} \\ -4(I) \\ -7(I) \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z & =0 \\ -3y-6z & =0 \\ -6y-11z & =0 \end{bmatrix} \div(-3) \rightarrow$

$$\begin{bmatrix} x+2y+3z & =0 \\ y+2z & =0 \\ -6y-11z & =0 \end{bmatrix} \begin{matrix} -2(II) \\ \\ +6(II) \end{matrix} \rightarrow \begin{bmatrix} x-z & =0 \\ y+2z & =0 \\ z & =0 \end{bmatrix} \begin{matrix} +III \\ -2(III) \\ \end{matrix} \rightarrow \begin{bmatrix} x & =0 \\ y & =0 \\ z & =0 \end{bmatrix},$$

so that $(x,y,z)=(0,0,0)$.

1.1.9 $\begin{bmatrix} x+2y+3z & =1 \\ 3x+2y+z & =1 \\ 7x+2y-3z & =1 \end{bmatrix} \begin{matrix} \\ -3(I) \\ -7(I) \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z & =1 \\ -4y-8z & =-2 \\ -12y-24z & =-6 \end{bmatrix} \div(-4) \rightarrow$

$$\begin{bmatrix} x+2y+3z & =1 \\ y+2z & =\frac{1}{2} \\ -12y-24z & =-6 \end{bmatrix} \begin{matrix} -2(II) \\ \\ +12(II) \end{matrix} \rightarrow \begin{bmatrix} x-z & =0 \\ y+2z & =\frac{1}{2} \\ 0 & =0 \end{bmatrix}$$

This system has infinitely many solutions: if we choose $z=t$, an arbitrary real number, then we get $x=z=t$ and $y=\frac{1}{2}-2z=\frac{1}{2}-2t$. Therefore, the general solution is $(x,y,z)=\left(t,\frac{1}{2}-2t,t\right)$, where t is an arbitrary real number.

1.1.10 $\begin{bmatrix} x+2y+3z & =1 \\ 2x+4y+7z & =2 \\ 3x+7y+11z & =8 \end{bmatrix} \begin{matrix} \\ -2(I) \\ -3(I) \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z & =1 \\ z & =0 \\ y+2z & =5 \end{bmatrix} \begin{matrix} Swap: \\ II \leftrightarrow III \end{matrix} \rightarrow$

$$\begin{bmatrix} x+2y+3z & =1 \\ y+2z & =5 \\ z & =0 \end{bmatrix} \begin{matrix} -2(II) \\ \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x-z & =-9 \\ y+2z & =5 \\ z & =0 \end{bmatrix} \begin{matrix} +III \\ -2(III) \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x & =-9 \\ y & =5 \\ z & =0 \end{bmatrix},$$

so that $(x,y,z)=(-9,5,0)$.

1.1.11 $\begin{bmatrix} x-2y & =2 \\ 3x+5y & =17 \end{bmatrix} \begin{matrix} \\ -3(I) \end{matrix} \rightarrow \begin{bmatrix} x-2y & =2 \\ 11y & =11 \end{bmatrix} \div 11 \rightarrow \begin{bmatrix} x-2y & =2 \\ y & =1 \end{bmatrix} \begin{matrix} +2(II) \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x & =4 \\ y & =1 \end{bmatrix},$

so that $(x,y)=(4,1)$. See Figure 1.1.

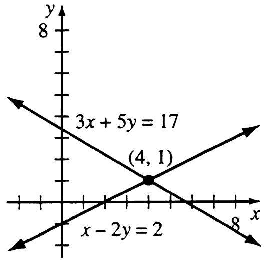

Figure 1.1: for Problem 1.1.11.

1.1.12 $\begin{bmatrix} x-2y & =3 \\ 2x-4y & =6 \end{bmatrix} \begin{matrix} \\ -2(I) \end{matrix} \rightarrow \begin{bmatrix} x-2y & =3 \\ 0 & =0 \end{bmatrix}$

This system has infinitely many solutions: If we choose $y = t$, an arbitrary real number, then the equation $x - 2y = 3$ gives us $x = 3 + 2y = 3 + 2t$. Therefore the general solution is $(x, y) = (3 + 2t, t)$, where t is an arbitrary real number. (See Figure 1.2.)

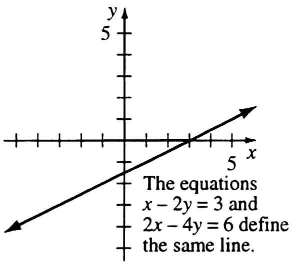

Figure 1.2: for Problem 1.1.12.

1.1.**13** $\begin{bmatrix} x - 2y & = 3 \\ 2x - 4y & = 8 \end{bmatrix} \begin{matrix} \\ -2(I) \end{matrix} \to \begin{bmatrix} x - 2y & = 3 \\ 0 & = 2 \end{bmatrix}$, which has no solutions. (See Figure 1.3.)

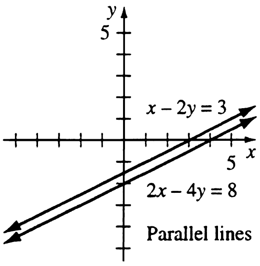

Figure 1.3: for Problem 1.1.13.

1.1.**14** The system reduces to $\begin{bmatrix} x + 5z & = 0 \\ y - z & = 0 \\ 0 & = 1 \end{bmatrix}$, so that there is no solution; no point in space belongs to all three planes.

Compare with Figure 2b.

1.1.**15** The system reduces to $\begin{bmatrix} x & = 0 \\ y & = 0 \\ z & = 0 \end{bmatrix}$ so the unique solution is $(x, y, z) = (0, 0, 0)$. The three planes intersect at the origin.

1.1.**16** The system reduces to $\begin{bmatrix} x + 5z & = 0 \\ y - z & = 0 \\ 0 & = 0 \end{bmatrix}$, so the solutions are of the form $(x, y, z) = (-5t, t, t)$, where t is an arbitrary number. The three planes intersect in a line; compare with Figure 2a.

1.1.**17** $\begin{bmatrix} x + 2y & = a \\ 3x + 5y & = b \end{bmatrix} \begin{matrix} \\ -3(I) \end{matrix} \to \begin{bmatrix} x + 2y & = a \\ -y & = -3a + b \end{bmatrix} \begin{matrix} \\ \div(-1) \end{matrix} \to \begin{bmatrix} x + 2y & = a \\ y & = 3a - b \end{bmatrix} \begin{matrix} -2(II) \\ \\ \end{matrix}$

$\begin{bmatrix} x & = -5a + 2b \\ y & = 3a - b \end{bmatrix}$, so that $(x, y) = (-5a + 2b,\ 3a - b)$.

1.1.**18** $\begin{bmatrix} x + 2y + 3z & = a \\ x + 3y + 8z & = b \\ x + 2y + 2z & = c \end{bmatrix} \begin{matrix} \\ -I \\ -I \end{matrix} \rightarrow \begin{bmatrix} x + 2y + 3z & = a \\ y + 5z & = -a + b \\ -z & = -a + c \end{bmatrix} \begin{matrix} -2(II) \\ \\ \\ \end{matrix} \rightarrow$

$\begin{bmatrix} x - 7z & = 3a - 2b \\ y + 5z & = -a + b \\ -z & = -a + c \end{bmatrix} \begin{matrix} \\ \\ \div(-1) \end{matrix} \rightarrow \begin{bmatrix} x - 7z & = 3a - 2b \\ y + 5z & = -a + b \\ z & = a - c \end{bmatrix} \begin{matrix} +7(III) \\ -5(III) \\ \\ \end{matrix} \rightarrow \begin{bmatrix} x & = 10a - 2b - 7c \\ y & = -6a + b + 5c \\ z & = a - c \end{bmatrix}$,

so that $(x, y, z) = (10a - 2b - 7c,\ -6a + b + 5c,\ a - c)$.

1.1.**19** a Note that the demand D_1 for product 1 increases with the increase of price P_2; likewise the demand D_2 for product 2 increases with the increase of price P_1. This indicates that the two products are competing; some people will switch if one of the products gets more expensive.

b Setting $D_1 = S_1$ and $D_2 = S_2$ we obtain the system $\begin{bmatrix} 70 - 2P_1 + P_2 & = -14 + 3P_1 \\ 105 + P_1 - P_2 & = -7 + 2P_2 \end{bmatrix}$, or $\begin{bmatrix} -5P_1 + P_2 & = -84 \\ P_1 - 3P_2 & = 112 \end{bmatrix}$, which yields the unique solution $P_1 = 26$ and $P_2 = 46$.

1.1.**20** The total demand for the product of Industry A is 1000 (the consumer demand) plus 0.1b (the demand from Industry B). The output a must meet this demand: $a = 1000 + 0.1b$.

Setting up a similar equation for Industry B we obtain the system $\begin{bmatrix} a & = 1000 + 0.1b \\ b & = 780 + 0.2a \end{bmatrix}$ or $\begin{bmatrix} a - 0.1b & = 1000 \\ -0.2a + b & = 780 \end{bmatrix}$, which yields the unique solution $a = 1100$ and $b = 1000$.

1.1.**21** The total demand for the products of Industry A is 310 (the consumer demand) plus $0.3b$ (the demand from Industry B). The output a must meet this demand: $a = 310 + 0.3b$.

Setting up a similar equation for Industry B we obtain the system $\begin{bmatrix} a & = 310 + 0.3b \\ b & = 100 + 0.5a \end{bmatrix}$ or $\begin{bmatrix} a - 0.3b & = 310 \\ -0.5a + b & = 100 \end{bmatrix}$, which yields the solution $a = 400$ and $b = 300$.

1.1.**22** Since $x(t) = a\sin(t) + b\cos(t)$ we can compute $\frac{dx}{dt} = a\cos(t) - b\sin(t)$ and $\frac{d^2x}{dt^2}$

$= -a\sin(t) - b\cos(t)$. Substituting these expressions into the equation $\frac{d^2x}{dt^2} - \frac{dx}{dt} - x = \cos(t)$ and simplifying gives $(b - 2a)\sin(t) + (-a - 2b)\cos(t) = \cos(t)$. Comparing the coefficients of $\sin(t)$ and $\cos(t)$ on both sides of the equation then yields the system $\begin{bmatrix} -2a + b & = 0 \\ -a - 2b & = 1 \end{bmatrix}$, so that $a = -\frac{1}{5}$ and $b = -\frac{2}{5}$. See Figure 1.4.

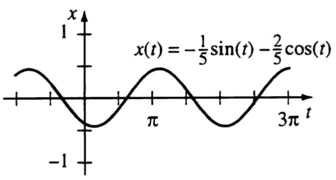

Figure 1.4: for Problem 1.1.22.

1.1.**23** a Substituting $\lambda = 5$ yields the system

$$\begin{bmatrix} 7x - y & = 5x \\ -6x + 8y & = 5y \end{bmatrix} \text{ or } \begin{bmatrix} 2x - y & = 0 \\ -6x + 3y & = 0 \end{bmatrix} \text{ or } \begin{bmatrix} 2x - y & = 0 \\ 0 & = 0 \end{bmatrix}.$$

There are infinitely many solutions, of the form $(x, y) = \left(\frac{t}{2}, t\right)$, where t is an arbitrary real number.

b Proceeding as in part (a), we find $(x, y) = \left(-\frac{1}{3}t, t\right)$.

c Proceedings as in part (a), we find only the solution $(0, 0)$.

1.1.**24** Let v be the speed of the boat relative to the water, and s be the speed of the stream; then the speed of the boat relative to the land is $v + s$ downstream and $v - s$ upstream. Using the fact that (distance) = (speed)(time), we obtain the system

$$\begin{bmatrix} 8 & = (v + s)\frac{1}{3} \\ 8 & = (v - s)\frac{2}{3} \end{bmatrix} \quad \begin{matrix} \leftarrow \text{ downstream} \\ \leftarrow \text{ upstream} \end{matrix}$$

The solution is $v = 18$ and $s = 6$.

1.1.**25** The system reduces to $\begin{bmatrix} x + z & = 1 \\ y - 2z & = -3 \\ 0 & = k - 7 \end{bmatrix}$.

a. The system has solutions if $k - 7 = 0$, or $k = 7$.

b. If $k = 7$ then the system has infinitely many solutions.

c. If $k = 7$ then we can choose $z = t$ freely and obtain the solutions

$(x, y, z) = (1 - t, -3 + 2t, t)$.

1.1.**26** The system reduces to $\begin{bmatrix} x - 3z & = & 1 \\ y + 2z & = & 1 \\ (k^2 - 4)z & = & k - 2 \end{bmatrix}$

This system has a unique solution if $k^2 - 4 \neq 0$, that is, if $k \neq \pm 2$.

If $k = 2$, then the last equation is $0 = 0$, and there will be infinitely many solutions.

If $k = -2$, then the last equation is $0 = -4$, and there will be no solutions.

1.1.**27** Let x = the number of male children and y = the number of female children.

Then the statement "Emile has twice as many sisters as brothers" translates into

$y = 2(x - 1)$ and "Gertrude has as many brothers as sisters" translates into

$x = y - 1$.

Solving the system $\begin{bmatrix} -2x + y & = -2 \\ x - y & = -1 \end{bmatrix}$ gives $x = 3$ and $y = 4$.

There are seven children in this family.

1.1.**28** The thermal equilibrium condition requires that $T_1 = \frac{T_2+200+0+0}{4}$, $T_2 = \frac{T_1+T_3+200+0}{4}$, and $T_3 = \frac{T_2+400+0+0}{4}$.

We can rewrite this system as $\begin{bmatrix} -4T_1 + T_2 & = -200 \\ T_1 - 4T_2 + T_3 & = -200 \\ T_2 - 4T_3 & = -400 \end{bmatrix}$

The solution is $(T_1, T_2, T_3) = (75, 100, 125)$.

1.1.**29** To assure that the graph goes through the point $(1, -1)$, we substitute $t = 1$ and $f(t) = -1$ into the equation $f(t) = a + bt + ct^2$ to give $-1 = a + b + c$.

Proceeding likewise for the two other points, we obtain the system $\begin{bmatrix} a + b + c & = -1 \\ a + 2b + 4c & = 3 \\ a + 3b + 9c & = 13 \end{bmatrix}$.

The solution is $a = 1,\ b = -5$, and $c = 3$, and the polynomial is $f(t) = 1 - 5t + 3t^2$. (See Figure 1.5.)

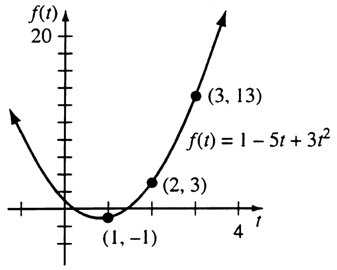

Figure 1.5: for Problem 1.1.29.

1.1.**30** Proceeding as in the previous exercise, we obtain the system $\left[\begin{array}{ll|} a + b + c & = p \\ a + 2b + 4c & = q \\ a + 3b + 9c & = r \end{array}\right.$.

The unique solution is $\begin{bmatrix} a & = 3p - 3q + r \\ b & = -2.5p + 4q - 1.5r \\ c & = 0.5p - q + 0.5r \end{bmatrix}$.

Only one polynomial of degree 2 goes through the three given points, namely,

$f(t) = 3p - 3q + r + (-2.5p + 4q - 1.5r)t + (0.5p - q + 0.5r)t^2$.

1.1.**31** $f(t)$ is of the form $at^2 + bt + c$. So $f(1) = a(1^2) + b(1) + c = 3$, and $f(2) = a(2^2) + b(2) + c = 6$. Also, $f'(t) = 2at + b$, meaning that $f'(1) = 2a + b = 1$.

So we have a system of equations: $\begin{bmatrix} a + b + c = 3 \\ 4a + 2b + c = 6 \\ 2a + b = 1 \end{bmatrix}$

which reduces to $\begin{bmatrix} a = 2 \\ b = -3 \\ c = 4 \end{bmatrix}$.

Thus, $f(t) = 2t^2 - 3t + 4$ is the only solution.

1.1.**32** $f(t)$ is of the form $at^2 + bt + c$. So, $f(1) = a(1^2) + b(1) + c = 1$ and $f(2) = 4a + 2b + c = 0$. Also, $\int_1^2 f(t)dt = \int_1^2 (at^2 + bt + c)dt$

$= \frac{a}{3}t^3 + \frac{b}{2}t^2 + ct|_1^2$

$= \frac{8}{3}a + 2b + 2c - (\frac{a}{3} + \frac{b}{2} + c)$

$= \frac{7}{3}a + \frac{3}{2}b + c = -1.$

So we have a system of equations: $\begin{bmatrix} a + b + c = 1 \\ 4a + 2b + c = 0 \\ \frac{7}{3}a + \frac{3}{2}b + c = -1 \end{bmatrix}$

which reduces to $\begin{bmatrix} a = 9 \\ b = -28 \\ c = 20 \end{bmatrix}$.

Thus, $f(t) = 9t^2 - 28t + 20$ is the only solution.

1.1.**33** $f(t)$ is of the form $at^2 + bt + c$. $f(1) = a + b + c = 1$, $f(3) = 9a + 3b + c = 3$, and $f'(t) = 2at + b$, so $f'(2) = 4a + b = 1$.

Now we set up our system to be $\begin{bmatrix} a + b + c = 1 \\ 9a + 3b + c = 3 \\ 4a + b = 1 \end{bmatrix}$.

This reduces to $\begin{bmatrix} a - \frac{c}{3} = 0 \\ b + \frac{4}{3}c = 1 \\ 0 = 0 \end{bmatrix}$.

We write everything in terms of a, revealing $c = 3a$ and $b = 1 - 4a$.

So, $f(t) = at^2 + (1 - 4a)t + 3a$ for an arbitrary a.

1.1.**34** $f(t) = at^2 + bt + c$, so $f(1) = a + b + c = 1, f(3) = 9a + 3b + c = 3$. Also, $f'(2) = 3$, so $2(2)a + b = 4a + b = 3$.

Thus, our system is $\begin{bmatrix} a + b + c = 1 \\ 9a + 3b + c = 3 \\ 4a + b = 3 \end{bmatrix}$.

When we reduce this, however, our last equation becomes $0 = 2$, meaning that this system is inconsistent.

1.1.**35** $f(t) = ae^{3t} + be^{2t}$, so $f(0) = a + b = 1$ and $f'(t) = 3ae^{3t} + 2be^{2t}$, so $f'(0) = 3a + 2b = 4$.

Thus we obtain the system $\begin{bmatrix} a + b = 1 \\ 3a + 2b = 4 \end{bmatrix}$,

which reveals $\begin{bmatrix} a = 2 \\ b = -1 \end{bmatrix}$.

So $f(t) = 2e^{3t} - e^{2t}$.

1.1.**36** $f(t) = a\cos(2t) + b\sin(2t)$ and $3f(t) + 2f'(t) + f''(t) = 17\cos(2t)$.

$f'(t) = 2b\cos(2t) - 2a\sin(2t)$ and $f''(t) = -4b\sin(2t) - 4a\cos(2t)$.

So, $17\cos(2t) = 3(a\cos(2t) + b\sin(2t)) + 2(2b\cos(2t) - 2a\sin(2t)) + (-4b\sin(2t) - 4a\cos(2t)) = (-4a + 4b + 3a)\cos(2t) + (-4b - 4a + 3b)\sin(2t) = (-a + 4b)\cos(2t) + (-4a - b)\sin(2t)$.

So, our system is: $\begin{bmatrix} -a + 4b = 17 \\ -4a - b = 0 \end{bmatrix}$.

This reduces to: $\begin{bmatrix} a = -1 \\ b = 4 \end{bmatrix}$.

So our function is $f(t) = -\cos(2t) + 4\sin(2t)$.

1.1.**37** Plugging the three points (x, y) into the equation $a + bx + cy + x^2 + y^2 = 0$, leads to a system of linear equations for the three unknowns (a, b, c).

$$\begin{aligned} a + 5b + 5c + 25 + 25 &= 0 \\ a + 4b + 6c + 16 + 36 &= 0 \\ a + 6b + 2c + 36 + 4 &= 0\ . \end{aligned}$$

The solution is $a = -20, b = -2, c = -4$. $-20 - 2x - 4y + x^2 + y^2 = 0$ is a circle of radius 5 centered at $(1, 2)$.

1.1.**38** Plug the three points into the equation $ax^2 + bxy + cy^2 = 1$. We obtain a system of linear equations

$$\begin{aligned} a + 2b + 4c &= 1 \\ 4a + 4b + 4c &= 1 \\ 9a + 3b + c &= 1\ . \end{aligned}$$

The solution is $a = 3/20, b = -9/40, c = 13/40$. This is the ellipse $(3/20)x^2 - (9/40)xy + (13/40)y^2 = 1$.

1.1.**39** The given system reduces to $\begin{bmatrix} x - z & = \frac{-5a+2b}{3} \\ y + 2z & = \frac{4a-b}{3} \\ 0 & = a - 2b + c \end{bmatrix}$.

This system has solutions (in fact infinitely many) if $a - 2b + c = 0$.

The points (a, b, c) with this property form a plane through the origin.

1.1.**40** a $x_1 = -3$

$x_2 = 14 + 3x_1 = 14 + 3(-3) = 5$

$x_3 = 9 - x_1 - 2x_2 = 9 + 3 - 10 = 2$

$x_4 = 33 + x_1 - 8x_2 + 5x_3 - x_4 = 33 - 3 - 40 + 10 = 0$,

so that $(x_1, x_2, x_3, x_4) = (-3, 5, 2, 0)$.

b $x_4 = 0$

$x_3 = 2 - 2x_4 = 2$

$x_2 = 5 - 3x_3 - 7x_4 = 5 - 6 = -1$

$x_1 = -3 - 2x_2 + x_3 - 4x_4 = -3 + 2 + 2 = 1,$

so that $(x_1, x_2, x_3, x_4) = (1, -1, 2, 0)$.

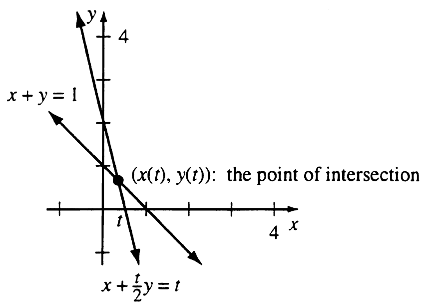

Figure 1.6: for Problem 1.1.41a.

1.1.**41** a The two lines intersect unless $t = 2$ (in which case both lines have slope -1).
To draw a rough sketch of $x(t)$, note that
$\lim_{t\to\infty} x(t) = \lim_{t\to-\infty} x(t) = -1$ $\big($the line $x + \frac{t}{2}y = t$ becomes almost horizontal$\big)$
and
$\lim_{t\to 2^-} x(t) = \infty$, $\lim_{t\to 2^+} x(t) = -\infty$.
Also note that $x(t)$ is positive if t is between 0 and 2, and negative otherwise.
Apply similar reasoning to $y(t)$. (See Figures 1.6 and 1.7.)

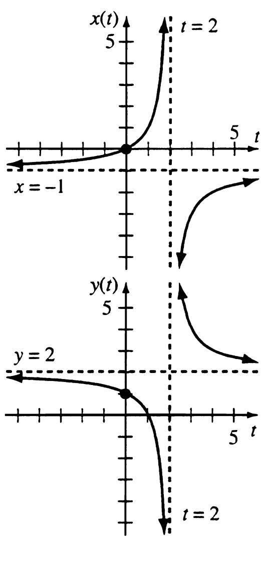

Figure 1.7: for Problem 1.1.41a.

b $x(t) = \frac{-t}{t-2}$, and $y(t) = \frac{2t-2}{t-2}$.

1.1.**42** We can think of the line through the points $(1,1,1)$ and $(3,5,0)$ as the intersection of any two planes through these two points; each of these planes will be defined by an equation of the form $ax+by+cz=d$. It is required that $1a+1b+1c=d$ and $3a+5b+0c=d$.

Now the system $\begin{bmatrix} a & +b & +c & -d & = & 0 \\ 3a & +5b & & -d & = & 0 \end{bmatrix}$ reduces to

$\begin{bmatrix} a & & +\frac{5}{2}c & -2d & = & 0 \\ & b & -\frac{3}{2}c & +d & = & 0 \end{bmatrix}$.

We can choose arbitrary real numbers for c and d; then $a = -\frac{5}{2}c + 2d$ and $b = \frac{3}{2}c - d$. For example, if we choose $c=2$ and $d=0$, then $a=-5$ and $b=3$, leading to the equation $-5x+3y+2z=0$. If we choose $c=0$ and $d=1$, then $a=2$ and $b=-1$, giving the equation $2x-y=1$.

We have found one possible answer: $\begin{bmatrix} -5x & +3y & +2z & = & 0 \\ 2x & -y & & = & 1 \end{bmatrix}$.

1.1.**43** To eliminate the arbitrary constant t, we can solve the last equation for t to give $t=z-2$, and substitute $z-2$ for t in the first two equations, obtaining $\begin{bmatrix} x & = 6+5(z-2) \\ y & = 4+3(z-2) \end{bmatrix}$ or $\begin{bmatrix} x-5z & = -4 \\ y-3z & = -2 \end{bmatrix}$.

This system does the job.

1.1.**44** Let b = Boris' money, m = Marina's money, and c = cost of a chocolate bar.

We are told that $\begin{bmatrix} b+\frac{1}{2}m & = c \\ \frac{1}{2}b+m & = 2c \end{bmatrix}$, with solution $(b,m)=(0,2c)$.

Boris has no money.

1.1.**45** Let us start by reducing the system:

$$\begin{bmatrix} x+2y+3z & = 39 \\ x+3y+2z & = 34 \\ 3x+2y+z & = 26 \end{bmatrix} \begin{matrix} \\ -I \\ -3(I) \end{matrix} \rightarrow \begin{bmatrix} x+2y+3z & = 39 \\ y-z & = -5 \\ -4y-8z & = -91 \end{bmatrix}$$

Note that the last two equations are exactly those we get when we substitute

$x=39-2y-3z$: either way, we end up with the system $\begin{bmatrix} y-z & = -5 \\ -4y-8z & = -91 \end{bmatrix}$.

1.1.**46** a We set up two equations here, with our variables: x_1 = servings of rice, x_2 = servings of yogurt.

So our system is: $\begin{bmatrix} 3x_1 & +12x_2 & = 60 \\ 30x_1 & +20x_2 & = 300 \end{bmatrix}$.

Solving this system reveals that $x_1=8, x_2=3$.

b Again, we set up our equations: $\begin{bmatrix} 3x_1 & +12x_2 & = P \\ 30x_1 & +20x_2 & = C \end{bmatrix}$,

and reduce them to find that $x_1 = -\frac{P}{15} + \frac{C}{25}$, while $x_2 = \frac{P}{10} - \frac{C}{100}$.

1.1.**47** Let x_1 = number of one-dollar bills, x_2 = the number of five-dollar bills, and x_3 = the number of ten-dollar bills. Then our system looks like: $\begin{bmatrix} x_1+x_2+x_3 & =32 \\ x_1+5x_2+10x_3 & =100 \end{bmatrix}$,

which reduces to give us solutions that fit: $x_1 = 15 + \frac{5}{4}x_3$, $x_2 = 17 - \frac{9}{4}x_3$, where x_3 can be chosen freely. Now let's keep in mind that x_1, x_2, and x_3 must be positive integers and see what conditions this imposes on the variable x_3. We see that since x_1 and x_2 must be integers, x_3 must be a multiple of 4. Furthermore, x_3 must be positive, and $x_2 = 17 - \frac{9}{4}x_3$ must be positive as well, meaning that $x_3 < \frac{68}{9}$. These constraints leave us with only one possibility, $x_3 = 4$, and we can compute the corresponding values $x_1 = 15 + \frac{5}{4}x_3 = 20$ and $x_2 = 17 - \frac{9}{4}x_3 = 8$.

Thus, we have 20 one-dollar bills, 8 five-dollar bills, and 4 ten-dollar bills.

1.1.**48** Let x_1, x_2, x_3 be the number of 20 cent, 50 cent, and 2 Euro coins, respectively. Then we need solutions to the system: $\begin{bmatrix} x_1 & +x_2 & +x_3 & =1000 \\ .2x_1 & +.5x_2 & +2x_3 & =1000 \end{bmatrix}$

this system reduces to: $\begin{bmatrix} x_1 & & -5x_3 & =-\frac{5000}{3} \\ & x_2 & +6x_3 & =\frac{8000}{3} \end{bmatrix}$.

Our solutions are then of the form $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5x_3 - \frac{5000}{3} \\ -6x_3 + \frac{8000}{3} \\ x_3 \end{bmatrix}$. Unfortunately for the meter maids, there are no integer solutions to this problem. If x_3 is an integer, then neither x_1 nor x_2 will be an integer, and no one will ever claim the Ferrari.

Section 1.2

1.2.**1** $\begin{bmatrix} 1 & 1 & -2 & \vdots & 5 \\ 2 & 3 & 4 & \vdots & 2 \end{bmatrix} \begin{matrix} \\ -2(I) \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & -2 & \vdots & 5 \\ 0 & 1 & 8 & \vdots & -8 \end{bmatrix} \begin{matrix} -II \\ \\ \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & -10 & \vdots & 13 \\ 0 & 1 & 8 & \vdots & -8 \end{bmatrix}$

$\begin{bmatrix} x-10z & =13 \\ y+8z & =-8 \end{bmatrix} \longrightarrow \begin{bmatrix} x & =13+10z \\ y & =-8-8z \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13+10t \\ -8-8t \\ t \end{bmatrix}$, where t is an arbitrary real number.

1.2.**2** $\begin{bmatrix} 3 & 4 & -1 & \vdots & 8 \\ 6 & 8 & -2 & \vdots & 3 \end{bmatrix} \begin{matrix} \div 3 \\ \\ \end{matrix} \rightarrow \begin{bmatrix} 1 & \frac{4}{3} & -\frac{1}{3} & \vdots & \frac{8}{3} \\ 6 & 8 & -2 & \vdots & 3 \end{bmatrix} \begin{matrix} \\ -6(I) \end{matrix} \rightarrow \begin{bmatrix} 1 & \frac{4}{3} & -\frac{1}{3} & \vdots & \frac{8}{3} \\ 0 & 0 & 0 & \vdots & -13 \end{bmatrix}$

This system has no solutions, since the last row represents the equation $0 = -13$.

1.2.**3** $x = 4 - 2y - 3z$

y and z are free variables; let $y = s$ and $z = t$.

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4-2s-3t \\ s \\ t \end{bmatrix}$, where s and t are arbitrary real numbers.

1.2.**4** $\begin{bmatrix} 1 & 1 \vdots & 1 \\ 2 & -1 \vdots & 5 \\ 3 & 4 \vdots & 2 \end{bmatrix} \begin{matrix} \\ -2(I) \\ -3(I) \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 \vdots & 1 \\ 0 & -3 \vdots & 3 \\ 0 & 1 \vdots & -1 \end{bmatrix} \div(-3) \rightarrow \begin{bmatrix} 1 & 1 \vdots & 1 \\ 0 & 1 \vdots & -1 \\ 0 & 1 \vdots & -1 \end{bmatrix} \begin{matrix} -II \\ \\ -II \end{matrix} \rightarrow$

$\begin{bmatrix} 1 & 0 \vdots & 2 \\ 0 & 1 \vdots & -1 \\ 0 & 0 \vdots & 0 \end{bmatrix}$, so that $\begin{matrix} x & = & 2 \\ y & = & -1 \end{matrix}$.

1.2.**5** $\begin{bmatrix} 0 & 0 & 1 & 1 \vdots & 0 \\ 0 & 1 & 1 & 0 \vdots & 0 \\ 1 & 1 & 0 & 0 \vdots & 0 \\ 1 & 0 & 0 & 1 \vdots & 0 \end{bmatrix} \begin{matrix} swap: \\ I \leftrightarrow III \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \vdots & 0 \\ 0 & 1 & 1 & 0 \vdots & 0 \\ 0 & 0 & 1 & 1 \vdots & 0 \\ 1 & 0 & 0 & 1 \vdots & 0 \end{bmatrix} \begin{matrix} \\ \\ \\ -I \end{matrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 0 \vdots & 0 \\ 0 & 1 & 1 & 0 \vdots & 0 \\ 0 & 0 & 1 & 1 \vdots & 0 \\ 0 & -1 & 0 & 1 \vdots & 0 \end{bmatrix} \begin{matrix} -II \\ \\ \\ +II \end{matrix} \rightarrow$

$\begin{bmatrix} 1 & 0 & -1 & 0 \vdots & 0 \\ 0 & 1 & 1 & 0 \vdots & 0 \\ 0 & 0 & 1 & 1 \vdots & 0 \\ 0 & 0 & 1 & 1 \vdots & 0 \end{bmatrix} \begin{matrix} +III \\ -III \\ \\ -III \end{matrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 1 \vdots & 0 \\ 0 & 1 & 0 & -1 \vdots & 0 \\ 0 & 0 & 1 & 1 \vdots & 0 \\ 0 & 0 & 0 & 0 \vdots & 0 \end{bmatrix}$

$\begin{bmatrix} x_1 & & & + & x_4 & = & 0 \\ & x_2 & & - & x_4 & = & 0 \\ & & x_3 & + & x_4 & = & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 & = -x_4 \\ x_2 & = x_4 \\ x_3 & = -x_4 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -t \\ t \\ -t \\ t \end{bmatrix}$, where t is an arbitrary real number.

1.2.**6** The system is in rref already.

$\begin{bmatrix} x_1 & = 3 + 7x_2 - x_5 \\ x_3 & = 2 + 2x_5 \\ x_4 & = 1 - x_5 \end{bmatrix}$

Let $x_2 = t$ and $x_5 = r$.

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 + 7t - r \\ t \\ 2 + 2r \\ 1 - r \\ r \end{bmatrix}$

1.2.**7** $\begin{bmatrix} 1 & 2 & 0 & 2 & 3 \vdots & 0 \\ 0 & 0 & 1 & 3 & 2 \vdots & 0 \\ 0 & 0 & 1 & 4 & -1 \vdots & 0 \\ 0 & 0 & 0 & 0 & 1 \vdots & 0 \end{bmatrix} \begin{matrix} \\ \\ -II \\ \\ \end{matrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 2 & 3 \vdots & 0 \\ 0 & 0 & 1 & 3 & 2 \vdots & 0 \\ 0 & 0 & 0 & 1 & -3 \vdots & 0 \\ 0 & 0 & 0 & 0 & 1 \vdots & 0 \end{bmatrix} \begin{matrix} -2(III) \\ -3(III) \\ \\ \\ \end{matrix} \rightarrow$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 9 \vdots & 0 \\ 0 & 0 & 1 & 0 & 11 \vdots & 0 \\ 0 & 0 & 0 & 1 & -3 \vdots & 0 \\ 0 & 0 & 0 & 0 & 1 \vdots & 0 \end{bmatrix} \begin{matrix} -9(IV) \\ -11(IV) \\ +3(IV) \end{matrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 0 & 0 \vdots & 0 \\ 0 & 0 & 1 & 0 & 0 \vdots & 0 \\ 0 & 0 & 0 & 1 & 0 \vdots & 0 \\ 0 & 0 & 0 & 0 & 1 \vdots & 0 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + 2x_2 & = 0 \\ x_3 & = 0 \\ x_4 & = 0 \\ x_5 & = 0 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 & = -2x_2 \\ x_3 & = 0 \\ x_4 & = 0 \\ x_5 & = 0 \end{bmatrix}$$

Let $x_2 = t$. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2t \\ t \\ 0 \\ 0 \\ 0 \end{bmatrix}$, where t is an arbitrary real number.

1.2.**8** $\begin{bmatrix} 0 & 1 & 0 & 2 & 3 \vdots & 0 \\ 0 & 0 & 0 & 4 & 8 \vdots & 0 \end{bmatrix} \div 4 \rightarrow \begin{bmatrix} 0 & 1 & 0 & 2 & 3 \vdots & 0 \\ 0 & 0 & 0 & 1 & 2 \vdots & 0 \end{bmatrix} -2(II) \begin{bmatrix} 0 & 1 & 0 & 0 & -1 \vdots & 0 \\ 0 & 0 & 0 & 1 & 2 \vdots & 0 \end{bmatrix}$

$$\begin{bmatrix} x_2 - x_5 & = 0 \\ x_4 + 2x_5 & = 0 \end{bmatrix} \longrightarrow \begin{bmatrix} x_2 & = x_5 \\ x_4 & = -2x_5 \end{bmatrix}$$

Let $x_1 = r,\ x_3 = s,\ x_5 = t$.

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} r \\ t \\ s \\ -2t \\ t \end{bmatrix}$, where r, t and s are arbitrary real numbers.

1.2.**9** $\begin{bmatrix} 0 & 0 & 0 & 1 & 2 & -1 \vdots & 2 \\ 1 & 2 & 0 & 0 & 1 & -1 \vdots & 0 \\ 1 & 2 & 2 & 0 & -1 & 1 \vdots & 2 \end{bmatrix} \begin{matrix} swap: \\ I \leftrightarrow II \end{matrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 \vdots & 0 \\ 0 & 0 & 0 & 1 & 2 & -1 \vdots & 2 \\ 1 & 2 & 2 & 0 & -2 & 1 \vdots & 2 \end{bmatrix} \begin{matrix} \\ \\ -I \end{matrix} \rightarrow$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 \vdots & 0 \\ 0 & 0 & 0 & 1 & 2 & -1 \vdots & 2 \\ 0 & 0 & 2 & 0 & -2 & 2 \vdots & 2 \end{bmatrix} \begin{matrix} swap: \\ II \leftrightarrow III \end{matrix} \rightarrow \begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 \vdots & 0 \\ 0 & 0 & 2 & 0 & -2 & 2 \vdots & 2 \\ 0 & 0 & 0 & 1 & 2 & -1 \vdots & 2 \end{bmatrix} \div 2 \rightarrow$$

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 \vdots & 0 \\ 0 & 0 & 1 & 0 & -1 & 1 \vdots & 1 \\ 0 & 0 & 0 & 1 & 2 & -1 \vdots & 2 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + 2x_2 + x_5 - x_6 & = 0 \\ x_3 - x_5 + x_6 & = 1 \\ x_4 + 2x_5 - x_6 & = 2 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 & = -2x_2 - x_5 + x_6 \\ x_3 & = 1 + x_5 - x_6 \\ x_4 & = 2 - 2x_5 + x_6 \end{bmatrix}$$

Let $x_2 = r,\ x_5 = s$, and $x_6 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -2r - s + t \\ r \\ 1 + s - t \\ 2 - 2s + t \\ s \\ t \end{bmatrix}$$, where r, s and t are arbitrary real numbers.

1.2.**10** The system reduces to $\begin{bmatrix} x_1 & & & + & x_4 & = & 1 \\ & x_2 & & - & 3x_4 & = & 2 \\ & & x_3 & + & 2x_4 & = & -3 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 & = 1 - x_4 \\ x_2 & = 2 + 3x_4 \\ x_3 & = -3 - 2x_4 \end{bmatrix}$

Let $x_4 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 - t \\ 2 + 3t \\ -3 - 2t \\ t \end{bmatrix}$$, where t is an arbitrary real number.

1.2.**11** The system reduces to $\begin{bmatrix} x_1 & & + & 2x_3 & & = & 0 \\ & x_2 & - & 3x_3 & & = & 4 \\ & & & & x_4 & = & -2 \end{bmatrix} \longrightarrow \begin{bmatrix} x_1 & = -2x_3 \\ x_2 & = 4 + 3x_3 \\ x_4 & = -2 \end{bmatrix}$.

Let $x_3 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2t \\ 4 + 3t \\ t \\ -2 \end{bmatrix}$$

1.2.**12** The system reduces to $\begin{bmatrix} x_1 & & & & + & 3.5x_5 & + & x_6 & = & 0 \\ & x_2 & & & + & x_5 & & & = & 0 \\ & & x_3 & & & & - & \frac{5}{3}x_6 & = & 0 \\ & & & x_4 & + & 3x_5 & + & x_6 & = & 0 \end{bmatrix} \longrightarrow$

$$\begin{bmatrix} x_1 & = -3.5x_5 - x_6 \\ x_2 & = -x_5 \\ x_3 & = \frac{5}{3}x_6 \\ x_4 & = -3x_5 - x_6 \end{bmatrix}.$$

Let $x_5 = r$ and $x_6 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} -3.5r - t \\ -r \\ \frac{5}{3}t \\ -3r - t \\ r \\ t \end{bmatrix}$$

1.2.**13** The system reduces to $\begin{bmatrix} x & & - & z & = & 0 \\ & y & + & 2z & = & 0 \\ & & & 0 & = & 1 \end{bmatrix}$.

There are no solutions.

1.2.**14** The system reduces to $\begin{bmatrix} x & + & 2y & & = & -2 \\ & & & z & = & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} x & = -2-2y \\ z & = 2 \end{bmatrix}$.

Let $y = t$.

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2-2t \\ t \\ 2 \end{bmatrix}$$

1.2.**15** The system reduces to $\begin{bmatrix} x & & & = & 4 \\ & y & & = & 2 \\ & & z & = & 1 \end{bmatrix}$.

1.2.**16** The system reduces to $\begin{bmatrix} x_1 + 2x_2 + 3x_3 & & +5x_5 & = & 6 \\ & x_4 & +2x_5 & = & 7 \end{bmatrix} \longrightarrow$

$\begin{bmatrix} x_1 & = 6 - 2x_2 - 3x_3 - 5x_5 \\ x_4 & = 7 - 2x_5 \end{bmatrix}$.

Let $x_2 = r,\ x_3 = s$, and $x_5 = t$.

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 6-2r-3s-5t \\ r \\ s \\ 7-2t \\ t \end{bmatrix}$$

1.2.**17** The system reduces to $\begin{bmatrix} x_1 & & & & & = & -\frac{8221}{4340} \\ & x_2 & & & & = & \frac{8591}{8680} \\ & & x_3 & & & = & \frac{4695}{434} \\ & & & x_4 & & = & -\frac{459}{434} \\ & & & & x_5 & = & \frac{699}{434} \end{bmatrix}$.

1.2.**18** a No, since the third column contains two leading ones.

b Yes

c No, since the third row contains a leading one, but the second row does not.

d Yes

1.2.**19** $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

1.2.**20** Four, namely $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & k \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (k is an arbitrary constant.)

1.2.**21** Four, namely $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & k \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ (k is an arbitrary constant.)

1.2.**22** Seven, namely $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & a & b \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & c \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 1 & 0 & d \\ 0 & 1 & e \end{bmatrix}$, $\begin{bmatrix} 1 & f & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Here, $a, b, \ldots, f$ are arbitrary constants.

1.2.**23** The conditions a, b, and c for the reduced row-echelon form correspond to the properties P1, P2, and P3 given on Page 13. The Gauss-Jordan algorithm, summarized on Page 15, guarantees that those properties are satisfied.

1.2.**24** Yes; each elementary row operation is reversible, that is, it can be "undone." For example, the operation of row swapping can be undone by swapping the same rows again. The operation of dividing a row by a scalar can be reversed by multiplying the same row by the same scalar.

1.2.**25** Yes; if A is transformed into B by a sequence of elementary row operations, then we can recover A from B by applying the inverse operations in the reversed order (compare with Exercise 24).

1.2.**26** Yes, by Exercise 25, since rref(A) is obtained from A by a sequence of elementary row operations.

1.2.**27** No; whatever elementary row operations you apply to $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, you cannot make the last column equal to zero.

1.2.**28** Suppose $(c_1, c_2, \ldots, c_n)$ is a solution of the system $\begin{bmatrix} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \cdots\cdots\cdots \end{bmatrix}$.

To keep the notation simple, suppose we add k times the first equation to the second; then the second equation of the new system will be $(a_{21} + ka_{11})x_1 + \cdots + (a_{2n} + ka_{1n})x_n = b_2 + kb_1$.

We have to verify that $(c_1, c_2, \ldots, c_n)$ is a solution of this new equation. Indeed, $(a_{21} + ka_{11})c_1 + \cdots + (a_{2n} + ka_{1n})c_n = a_{21}c_1 + \cdots + a_{2n}c_n + k(a_{11}c_1 + \cdots + a_{1n}c_n) = b_2 + kb_1$.

We have shown that any solution of the "old" system is also a solution of the "new." To see that, conversely, any solution of the new system is also a solution of the old system, note that elementary row operations are reversible (compare with Exercise 24); we can obtain the old system by subtracting k times the first equation from the second equation of the new system.

1.2.**29** Since the number of oxygen atoms remains constant, we must have $2a + b = 2c + 3d$.

Considering hydrogen and nitrogen as well, we obtain the system $\begin{bmatrix} 2a & + & b & = & 2c & + & 3d \\ & & 2b & = & c & + & d \\ a & & & = & c & + & d \end{bmatrix}$ or

$\begin{bmatrix} 2a & + & b & - & 2c & - & 3d & = & 0 \\ & & 2b & - & c & - & d & = & 0 \\ a & & & - & c & - & d & = & 0 \end{bmatrix}$, which reduces to $\begin{bmatrix} a & & & - & 2d & = & 0 \\ & b & & - & d & = & 0 \\ & & c & - & d & = & 0 \end{bmatrix}$.

The solutions are $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 2t \\ t \\ t \\ t \end{bmatrix}$.

To get the smallest positive integers, we set $t = 1$:

$2NO_2 + H_2O \longrightarrow HNO_2 + HNO_3$

1.2.**30** Plugging the points into $f(t)$, we obtain the system

$$\begin{bmatrix} a & & & & & & & = & 1 \\ a & + & b & + & c & + & d & = & 0 \\ a & - & b & + & c & - & d & = & 0 \\ a & + & 2b & + & 4c & + & 8d & = & -15 \end{bmatrix}$$

with unique solution $a = 1,\ b = 2,\ c = -1$, and $d = -2$, so that $f(t) = 1 + 2t - t^2 - 2t^3$. (See Figure 1.8.)

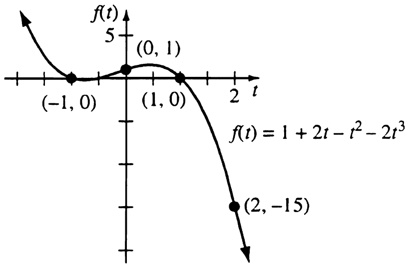

Figure 1.8: for Problem 1.2.30.

1.2.**31** Let $f(t) = a + bt + ct^2 + dt^3 + et^4$. Substituting the points in, we get

$$\begin{bmatrix} a & + & b & + & c & + & d & + & e & = & 1 \\ a & + & 2b & + & 4c & + & 8d & + & 16e & = & -1 \\ a & + & 3b & + & 9c & + & 27d & + & 81e & = & -59 \\ a & - & b & + & c & - & d & + & e & = & 5 \\ a & - & 2b & + & 4c & - & 8d & + & 16e & = & -29 \end{bmatrix}$$

This system has the unique solution $a = 1,\ b = -5,\ c = 4,\ d = 3$, and $e = -2$, so that $f(t) = 1 - 5t + 4t^2 + 3t^3 - 2t^4$. (See Figure 1.9.)

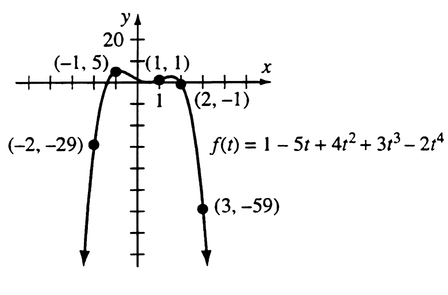

Figure 1.9: for Problem 1.2.31.