

PAUL A. FOERSTER

Calculus

Concepts and Applications



Key Curriculum Press®

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Second Edition



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Innovators in Mathematics Education

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To people from the past, including James H. Marable of Oak Ridge National Laboratory, from whom I first understood the concepts of calculus; Edmund Eickenroht, my former student, whose desire it was to write his own calculus text; and my late wife, Jo Ann.

To my wife, Peggy, who shares my zest for life and accomplishment.

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This text was written during the period when graphing calculator technology was making radical changes in the teaching and learning of calculus. The fundamental differences embodied in the text have arisen from teaching my own students using this technology. In addition, the text has been thoroughly revised to incorporate comments and suggestions from the many consultants and field testers listed on the previous page.

Thanks in particular to the original field test people—Betty Baker, Chris Comins, Debbie Davies, Val Guernon, David Heckman, Don Hight, Kathy Layton, Guy Mauldin, Windle McKenzie, Debbie Preston, Gary Starr, and John Wojtowicz. These instructors were enterprising enough to venture into a new approach to teaching calculus and to put up with the difficulties of receiving materials at the last minute.

Special thanks to Bill Medigovich for editing the first edition, coordinating the field test program, and organizing the first two summer institutes for instructors. Special thanks also to Debbie Preston for drafting the major part of the *Instructor's Guide* and parts of the *Solutions Manual*, and for working with the summer institutes for instructors. By serving as both instructors and consultants, these two have given this text an added dimension of clarity and teachability.

Thanks also to my students for enduring all those handouts, and for finding things to be changed! Special thanks to my students Craig Browning, Meredith Fast, William Fisher, Brad Wier, and Matthew Willis for taking good class notes so that the text materials could include classroom-tested examples.

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Finally, thanks to Chris Sollars, Debbie Davies, and Debbie Preston for their ideas and encouragement as I worked on the second edition of *Calculus*.

Paul A. Foerster

About the Author

Paul Foerster enjoys teaching mathematics at Alamo Heights High School in San Antonio, Texas, which he has done since 1961. After earning a bachelor's degree in chemical engineering, he served four years in the U.S. Navy. Following his first five years at Alamo Heights, he earned a master's degree in mathematics. He has published five textbooks, based on problems he wrote for his own students to let them see more realistically how mathematics is applied in the real world. In 1983 he received the Presidential Award for Excellence in Mathematics Teaching, the first year of the award. He raised three children with the late Jo Ann Foerster, and he also has two grown stepchildren through his wife Peggy Foerster, as well as three grandchildren. Paul plans to continue teaching for the foreseeable future, relishing the excitement of the ever-changing content of the evolving mathematics curriculum.

Foreword

by John Kenelly, Clemson University

In the explosion of the information age and the resulting instructional reforms, we have all had to deal repeatedly with the question: “When machines do mathematics, what do mathematicians do?” Many feel that our historical role has *not* changed, but that the emphasis is now clearly on selection and interpretation rather than manipulation and methods. As teachers, we continue to sense the need for a major shift in the instructional means we employ to impart mathematical understanding to our students. At the same time, we recognize that behind any technology there must be human insight.

In a world of change, we must build on the past and take advantage of the future. Applications and carefully chosen examples still guide us through what works. Challenges and orderly investigations still develop mature thinking and insights. As much as the instructional environment might change, quality education remains our goal. What we need are authors and texts that bridge the transition. It is in this regard that Paul Foerster and his texts provide outstanding answers.

In *Calculus: Concepts and Applications*, Second Edition, Paul is again at his famous best. The material is presented in an easily understood fashion with ample technology-based examples and exercises. The applications are intimately connected with the topic and amplify the key elements in the section. The material is a wealth of both fresh items and ancient insights that have stood the test of time. For example, alongside Escalante’s “cross hatch” method of repeated integration by parts, you’ll find Heaviside’s thumb trick for solving partial fractions! The students are repeatedly sent to their “graphers.” Early on, when differentiation is introduced, Paul discusses local linearity, and later he utilizes the zoom features of calculators in the coverage of l’Hospital’s rule—that’s fresh. Later still, he presents the logistic curve and slope fields in differential equations. All of these are beautiful examples of how computing technology has changed the calculus course.

The changes and additions found in this second edition exhibit the timeliness of the text. Exponentials and logarithms have been given an even more prominent role that reflects their greater emphasis in today’s calculus instruction. The narrative, problem sets, Explorations, and tests all support the position that the

choice between technology and traditional methods is not exclusively “one or the other” but *correctly* both. Rich, substantive, in-depth questions bring to mind superb Advanced Placement free response questions, or it might be that many AP questions remind you of Foerster’s style!

Throughout, you see how comprehensive Paul is in his study of the historical role of calculus and the currency of his understanding of the AP community and collegiate “calculus reform.” Brilliant, timely, solid, and loaded with tons of novel applications—your typical Foerster!

John Kenelly has been involved with the Advanced Placement Calculus program for over 30 years. He was Chief Reader and later Chair of the AP Calculus Committee when Paul Foerster was grading the AP exams in the 1970s. He is a leader in development of the graphing calculator and in pioneering its use in college and school classrooms. He served as president of the IMO 2001 USA, the organization that acts as host when the International Mathematical Olympiad (IMO) comes to the United States.

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