

**CliffsNotes®**  
**Parent's**  
***Crash Course***

ELEMENTARY SCHOOL  
**MATH**

David Alan Herzog



WILEY


Wiley Publishing, Inc.



**CliffsNotes®**  
**Parent's**  
***Crash Course***

ELEMENTARY SCHOOL  
**MATH**





**CliffsNotes®**  
**Parent's**  
***Crash Course***

ELEMENTARY SCHOOL  
**MATH**

David Alan Herzog



WILEY

Wiley Publishing, Inc.

## CliffsNotes® Parent's Crash Course Elementary School Math

Published by:  
Wiley Publishing, Inc.  
111 River Street  
Hoboken, NJ 07030-5774  
www.wiley.com

Copyright © 2005 by Wiley Publishing, Inc. All rights reserved.

Published simultaneously in Canada

No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 646-8600, or on the web at [www.copyright.com](http://www.copyright.com). Requests to the Publisher for permission should be addressed to the Legal Department, Wiley Publishing, Inc., 10475 Crosspoint Blvd., Indianapolis, IN 46256, (317) 572-3447, fax (317) 572-4355, or online at <http://www.wiley.com/go/permissions>.

**Trademarks:** Wiley, the Wiley Publishing logo, CliffsNotes, cliffsnotes.com, and all related trademarks, logos, and trade dress are trademarks or registered trademarks of John Wiley & Sons, Inc. and/or its affiliates and may not be used without written permission. [Insert third-party trademarks from book title or included logos here]. All other trademarks are the property of their respective owners. Wiley Publishing, Inc. is not associated with any product or vendor mentioned in this book.

The publisher and the author make no representations or warranties with respect to the accuracy or completeness of the contents of this work and specifically disclaim all warranties, including without limitation warranties of fitness for a particular purpose. No warranty may be created or extended by sales or promotional materials. The advice and strategies contained herein may not be suitable for every situation. This work is sold with the understanding that the publisher is not engaged in rendering legal, accounting, or other professional services. If professional assistance is required, the services of a competent professional person should be sought. Neither the publisher nor the author shall be liable for damages arising here from. The fact that an organization or Website is referred to in this work as a citation and/or a potential source of further information does not mean that the author or the publisher endorses the information the organization or Website may provide or recommendations it may make. Further, readers should be aware that Internet Websites listed in this work may have changed or disappeared between when this work was written and when it is read.

For general information on our other products and services or to obtain technical support please contact our Customer Care Department within the U.S. at (800) 762-2974, outside the U.S. at (317) 572-3993 or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic books. For more information about Wiley products, please visit our web site at [www.wiley.com](http://www.wiley.com).

Library of Congress Cataloging-in-Publication Data:

Herzog, David Alan.

CliffsNotes parents' crash course elementary school math / David A. Herzog.

p. cm.

ISBN-13: 978-0-7645-9836-4 (pbk.)

ISBN-10: 0-7645-9836-8

1. Mathematics--Study and teaching (Elementary). I. Title: Eelmentary school math. II. Title

QA135.6.H47 2006

510--dc22

2005018890

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

Book design by Elizabeth Brooks

Cover design by Jose Almaguer

Book production by Wiley Publishing, Inc. Composition Services

*This book is dedicated to my sister Lois who has successfully undergone surgeries for a brain tumor, several melanomas, breast cancer twice, and thyroid cancer, and keeps on plugging along as an inspiration for all of us.*

*It is also dedicated to Uncles Ian and Dylan, Alex and Jakob Cherry, Reese, Myles, Kira, and Rocio Herzog, Hailee Foster, Gino Nicholas, Sebastian, and Francesco Bubba, all their parents, and Grandma Birdie.*

## **Acknowledgment**

---

The late Robert B. Davis, professor at Webster College and Syracuse University, was director of the Madison Project in the 1960s and 1970s, a program named for the Madison School in Webster Grove, Missouri, primarily funded by the National Science Foundation, and seeking to improve the teaching of school mathematics.

Bob, as he was known to almost everyone who was acquainted with him, was a quiet, dedicated man, who worked tirelessly to help math teachers to make their subject more user-friendly to their pupils. He was widely responsible for familiarizing the rest of us with the work of Jean Piaget, and to the U.K.'s Nuffield project. Through a series of freely lent films of himself and other selected teachers working innovatively in the classroom, I and others gained a clearer understanding of how math can be taught as a subject our students could enjoy learning.

I was honored to have been taken under his wing in the early seventies, as an associate of the Madison Project, while participating in one of his workshops at Syracuse University, and I shall be ever grateful for his influence on my subsequent successes as an educator.

# Table of Contents

---

Introduction: Combining and Uncombining . . . . .	xii
<b>PART 1: WHOLE NUMBERS . . . . .</b>	<b>3</b>
Terms You Should Know . . . . .	4
Lesson 1: Order and Magnitude . . . . .	6
Lesson 2: Addition Facts . . . . .	8
Lesson 3: Subtraction Facts . . . . .	10
Lesson 4: Multiplication Facts . . . . .	12
Lesson 5: Division Facts . . . . .	14
Lesson 6: Place Value . . . . .	16
Lesson 7: Place Value Addition . . . . .	21
Lesson 8: Place Value Subtraction . . . . .	28
Lesson 9: Place Value Multiplication . . . . .	34
One-Digit by Two-Digit . . . . .	34
Two-Digit by Two-Digit . . . . .	35
Two-Digit by Three-Digit . . . . .	36
Three-Digit by Three-Digit . . . . .	36
Lesson 10: Place Value Division . . . . .	40
Remainders . . . . .	42
Estimating Quotients . . . . .	43
Lesson 11: Adding and Subtracting Larger Numbers . . . . .	46
Estimating for Addition . . . . .	46
Estimating for subtraction . . . . .	47
Second Chances . . . . .	48
Lesson 12: Multiplying Larger Numbers . . . . .	52
Mental Multiplication . . . . .	52
Estimating Large Products . . . . .	52
Lesson 13: Dividing Large Numbers . . . . .	57
Mental Division . . . . .	57
Short Division . . . . .	58
Estimating Large Quotients . . . . .	59



---

<b>Lesson 14: Evens, Primes, and Divisibility</b> .....	<b>62</b>
Prime Numbers .....	62
Divisibility Tests .....	63
<b>PART 2: FRACTIONS</b> .....	<b>67</b>
<b>Terms You Should Know</b> .....	<b>68</b>
<b>Lesson 15: Common Fractions</b> .....	<b>70</b>
Fraction as Part of a Whole .....	70
Fraction as Part of a Group .....	72
Fraction as a Whole .....	73
Fraction as a Comparison .....	74
Fraction as a Division .....	74
Proper versus Improper .....	75
<b>Lesson 16: Equivalent Fractions</b> .....	<b>76</b>
Common Equivalent Fractions .....	76
Creating Equivalent Fractions .....	78
<b>Lesson 17: Expressing Fractions in Lowest Terms</b> .....	<b>80</b>
GCF (Greatest Common Factor) .....	80
Reducing .....	81
<b>Lesson 18: Adding Fractions</b> .....	<b>83</b>
Common Multiples .....	83
Factor Trees Return .....	85
<b>Lesson 19: Subtracting Fractions</b> .....	<b>88</b>
<b>Lesson 20: Multiplying Fractions</b> .....	<b>90</b>
<b>Lesson 21: Dividing Fractions</b> .....	<b>92</b>
Reciprocals .....	92
Reciprocal Multiplication .....	92
<b>Lesson 22: Canceling</b> .....	<b>96</b>
Multiplying Fractions .....	96
Dividing Fractions .....	97

<b>Lesson 23: Mixed Numbers</b> .....	<b>100</b>
<b>Lesson 24: Adding Mixed Numbers</b> .....	<b>102</b>
With Unlike Denominators .....	102
Three or More .....	103
<b>Lesson 25: Subtracting Mixed Numbers</b> .....	<b>106</b>
Those that Work As Expected .....	106
The Larger Fraction's in the Denominator .....	106
<b>Lesson 26: Multiplying and Dividing Mixed Numbers</b> .....	<b>110</b>
Multiplying Mixed Numbers .....	110
Dividing Mixed Numbers .....	111
<b>Lesson 27: Decimal Fractions</b> .....	<b>114</b>
Extending Place Value .....	114
The Place-Holder .....	115
Some Common Decimal Fractions .....	115
Money .....	116
<b>Lesson 28: Adding Decimals</b> .....	<b>118</b>
A Jagged Affair .....	118
Three and More Numbers .....	119
<b>Lesson 29: Subtracting Decimals</b> .....	<b>120</b>
The Obvious Part .....	120
The Less Obvious Part .....	120
<b>Lesson 30: Multiplying Decimals</b> .....	<b>123</b>
<b>Lesson 31: Dividing Decimals</b> .....	<b>125</b>
<b>Lesson 32: Percent Fractions</b> .....	<b>127</b>
Common Fraction to Percent .....	128
Percent to Common Fraction .....	128
Decimal Fraction to Percent .....	129
Percent to Decimal Fraction .....	129
Decimals, Common Fractions, and Percents .....	129
<b>Lesson 33: The Arithmetic of Percents</b> .....	<b>132</b>
Taking a Percent of Something .....	132
Consecutive Discounts .....	133

---

Finding the Percentage . . . . .	133
Determining the Original . . . . .	134
<b>PART 3: SOME ESOTERIC STUFF . . . . .</b>	<b>137</b>
<b>Terms You Should Know . . . . .</b>	<b>138</b>
<b>Lesson 34: Averages . . . . .</b>	<b>139</b>
Arithmetic Mean . . . . .	139
Median . . . . .	140
Mode . . . . .	141
<b>Lesson 35: Integers . . . . .</b>	<b>143</b>
<b>Lesson 36: Adding Integers . . . . .</b>	<b>146</b>
Absolute Value . . . . .	146
Positive Plus Positive . . . . .	146
Negative Plus Negative . . . . .	147
Positive Plus Negative . . . . .	147
Three or More Addends . . . . .	148
<b>Lesson 37: Subtracting Integers . . . . .</b>	<b>150</b>
Subtracting a Positive . . . . .	152
Subtracting a Negative . . . . .	152
<b>Lesson 38: Multiplying and Dividing Integers . . . . .</b>	<b>154</b>
Like Signs . . . . .	154
Unlike Signs . . . . .	154
<b>Lesson 39: Introducing Exponents . . . . .</b>	<b>156</b>
Squares and Cubes . . . . .	156
Extending the Pattern . . . . .	156
<b>Lesson 40: More About Exponents . . . . .</b>	<b>158</b>
Multiplying and Dividing Exponential Terms . . . . .	158
Negative Exponents . . . . .	159
The Zeroth Power . . . . .	159
Place-Headings as Powers of 10 . . . . .	159

<b>Lesson 41: Scientific Notation</b> .....	<b>162</b>
Numbers Greater than One .....	162
Numbers Less than One .....	163
Undoing Scientific Notation .....	164
<b>Lesson 42: Roots</b> .....	<b>166</b>
Perfect Squares .....	166
Simplifying under the Radical Sign .....	167
Cube Roots .....	167
nth Roots .....	168

## **PART 4: GEOMETRY AND MEASUREMENT** .....

<b>Terms You Should Know</b> .....	<b>172</b>
<b>Lesson 43: Traditional versus SI Units</b> .....	<b>175</b>
Traditional Units of Measure .....	175
SI Units (Often Called “the Metric System”) .....	177
<b>Lesson 44: SI Prefixes</b> .....	<b>180</b>
Fractional Prefixes .....	180
Multiplier Prefixes .....	180
Converting SI Units .....	181
<b>Lesson 45: Linear Measure</b> .....	<b>182</b>
<b>Lesson 46: Mass and Weight</b> .....	<b>185</b>
<b>Lesson 47: Temperature</b> .....	<b>187</b>
Fahrenheit .....	187
Celsius, Once Called Centigrade .....	187
Kelvin a.k.a. Absolute .....	188
Converting to and from Fahrenheit .....	189
<b>Lesson 48: Area</b> .....	<b>192</b>
Quadrilaterals .....	192
Triangles .....	194
<b>Lesson 49: Circles</b> .....	<b>198</b>
Lines and Segments Related to Circles .....	198
Perimeter of a Circle .....	199
Area of a Circle .....	201

---

<b>Lesson 50: A Solid Start</b> . . . . .	<b>203</b>
Surface Area . . . . .	203
Volume of a Cube . . . . .	206
<b>Lesson 51: Prisms</b> . . . . .	<b>208</b>
Surface Areas of Right Prisms . . . . .	208
Volume of a Prism . . . . .	210
<b>Lesson 52: Solid Capacity</b> . . . . .	<b>214</b>
Traditional Measures of Capacity . . . . .	214
SI Units of Capacity . . . . .	215
<b>Lesson 53: Cylinders</b> . . . . .	<b>218</b>
Surface Area of a Cylinder . . . . .	218
Volume of a Cylinder . . . . .	219
<b>Lesson 54: Pyramids, Cones, and Spheres</b> . . . . .	<b>221</b>
Pyramids . . . . .	221
Cones . . . . .	224
Spheres . . . . .	225
<b>Lesson 55: Minding One’s Ps and Qs</b> . . . . .	<b>228</b>
<b>Lesson 56: Liter, Not Litter</b> . . . . .	<b>231</b>
<b>Lesson 57: FOO</b> . . . . .	<b>233</b>
<b>Lesson 58: Probability</b> . . . . .	<b>235</b>
Tossing a Coin . . . . .	235
Spinners . . . . .	236
Dependent Probabilities . . . . .	238
<b>APPENDIX: MORE HELP FOR YOUR STUDENT</b> . . . . .	<b>241</b>
Postman Stories . . . . .	242
Avogadro’s Number . . . . .	243
Flash Card Templates for Multiplication and Division . . . . .	243

# Introduction: Combining and Uncombining

Often parents are called upon to help their school-aged children with their homework, and just as often the parents have been removed from the subject matter for so long that they need to refamiliarize themselves with the basics of the material. That's what the *Parent's Crash Course* series is about.

This particular book is intended to help you, as parents, refresh your understanding of and familiarity with elementary school mathematics. Elementary math teachers approach the subject in several different ways, depending upon the way they were trained to teach it and their personal educational philosophies. Although we can't hope to cover all of those philosophies and techniques in the scope of this one volume, we'll try to stick to the underlying subject matter, so that you'll have the information that you need which you can then plug into the particular teaching approach being used at your child's school.

The concept of number is probably the first topic that needs to be dealt with, but we'll take that up second. Before that we need to take an overview of arithmetic—the primary operations of mathematics. There are two and only two types of operations in arithmetic. First is the type of operation that combines numbers, or puts them together. Second is the type of operation that uncombines numbers, or takes them apart.

When it comes to combining numbers, we can use two sub operations. The first is addition, wherein numbers are combined by a method that can be as simple as tallying. You have two sheep; you make two marks. You get three more sheep; make three more marks. How many sheep do you have altogether? Well, to find out, count up the marks. This is known as placing sheep and marks in one-to-one correspondence. This probably was the earliest form of numeration and counting used by the first shepherds.

Primitive tribes used to have separate sets of counters for tallying flat things and round things. By the time numbers tallied got much larger than five, the idea of four marks and a slash to mark the complete five came into favor. After all, this is not much different than counting on one's fingers, and grouping the amounts into fives, just like the number of fingers on each hand. Try not to lose sight of the fact that the main reason we count in a system based on the number 10 is because we have two hands, and a total of 10 fingers on them.

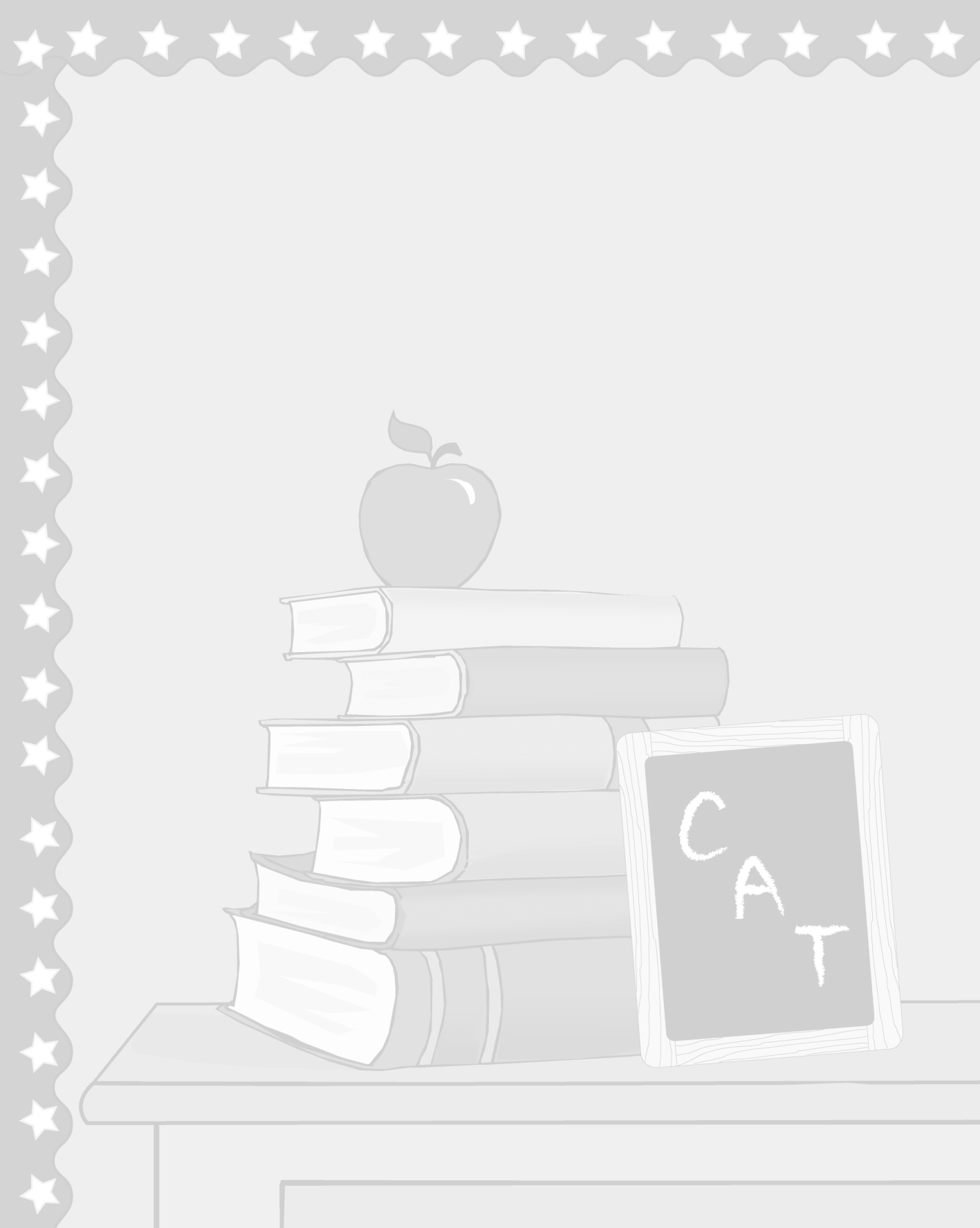
Of course, our system of numbers has made the process in some ways more complicated, and in some ways easier. The second combining operation is multiplication. Multiplying is really a shortcut for repeated adding of the same quantity, for example  $3 + 3 + 3 + 3$  can be represented by  $4 \times 3$  (4 times 3). The results are the same, 12.

For uncombining, we have the operation of subtraction, denoted by the minus sign ( $-$ ). If we started out with 16 goats and 4 of them were sold, we remove those sold goats from our flock:  $16 - 4 = 12$ . Take away 4 goats from 16 and only 12 goats **remain**. (You'll find words appearing in **boldface** in each chapter are in the Terms You Should Know section at the beginning of each Part.) The other uncombining operation is division. Often students have difficulty understanding division because they don't get the whole story. Division is repeated subtraction of the same number.  $16 - 4 - 4 - 4 - 4$ . Having reached 0,

we now count up how many 4s we were able to subtract and discover that it was 4. That might be written as  $16 \div 4$ , or 16 divided by 4. Division is also the undoing of multiplication.  $16 \times \_ = 4$  could also be written  $16 \div 4 = \_$ . In all cases, the quotient is 4.

One thing is very important to bear in mind when learning or helping one to learn mathematics: Math is, in the immortal word of *Star Trek*'s Mr. Spock, "logical." If what you have just read does not make sense to you, read it again. The odds are that you missed or misread something. If after thinking about and trying out what you've read, it still doesn't make sense, go online if you have access to the Internet. There are plenty of math-help sites out there. Otherwise, call a friend who might have a better knowledge of the subject matter and can explain it to you. After all, if you don't understand something, you're not going to be able to help your student to understand it. Hopefully, I've made things clear enough in this book that you'll understand what each topic is about, but never hesitate to question.

Now that we've gotten that out of the way, let's get down to the concept of whole numbers.





# Part 1

## Whole Numbers

	Terms You Should Know . . . . .	.4
1	Order and Magnitude . . . . .	.6
2	Addition Facts . . . . .	.8
3	Subtraction Facts . . . . .	.10
4	Multiplication Facts . . . . .	.12
5	Division Facts . . . . .	.14
6	Place Value . . . . .	.16
7	Place Value Addition . . . . .	.21
8	Place Value Subtraction . . . . .	.28
9	Place Value Multiplication . . . . .	.34
10	Place Value Division . . . . .	.40
11	Adding and Subtracting Larger Numbers . . . . .	.46
12	Multiplying Larger Numbers . . . . .	.52
13	Dividing Larger Numbers . . . . .	.57
14	Evens, Primes, and Divisibility . . . . .	.62



# Terms You Should Know

After each word in the Glossary, the lesson where it first appears is cited. Occasionally a second or third appearance is also given if there are additional major references, but not all appearances of each word are necessarily cited.

**append** (Lesson 12). Stick on to the end; Example: to multiply 43 by 100, append two zeroes to the 43. [ $43 \times 100 = 4300$ ].

**associative property** (Lessons 2, 4). Applies to addition and multiplication only and says however you group the numbers for addition or multiplication, the answer is not affected.

**cardinal numbers** (Lesson 1). The numbers we count with; a.k.a. counting numbers or natural numbers.

**commutative property for addition** (Lesson 2). No matter which way you add two numbers, the answer is always the same. That may be abbreviated as  $a + b = b + a = c$ .

**commutative property for multiplication** (Lesson 4). No matter which way you multiply two numbers, the answer is always the same. That may be abbreviated as  $a \times b = b \times a = c$ .

**composite number** (Lesson 14). The name applied to a number with more than 2 factors.

**counting numbers** (Lesson 1). See cardinal numbers.

**decade** (Lesson 13). A group of ten things; another name for the tens place (in addition, of course, to being ten years).

**difference** (Lesson 3). The answer in a subtraction; see remainder.

**dividend** (Lesson 10). In a division, the number being divided into.

**divisible** (Lesson 14). Can be perfectly divided by another.

**divisor** (Lesson 5). The number being divided by in a division.

**exchange** (Lessons 2, 3). See “rename”; also used to refer to money or barter.

**factor** (Lessons 9, 12, 14). **1. *n.*** A number multiplied to form another. **2. *v.t.*** To divide a quantity out of another.

**ladder division** (Lesson 10). A form of division in which groups of the divisor are repeatedly subtracted and written on steps below the division bracket.

**magnitude** (Lesson 1). A fancy word for size.

**minuend** (Lesson 8). The top number in a place value subtraction, or the number being subtracted from.

**multipliland** (Lesson 9). The number being multiplied; the top number in a place value multiplication.

**multiplier** (Lesson 9). The number being multiplied by; the bottom number in a place value multiplication.

**natural numbers** (Lesson 1). See cardinal numbers.

**ordinal numbers** (Lesson 1). The numbers that are used to show position, that is 1st, 2nd, 3rd, . . . 25th, and so on.

**parameter** (Lesson 13). Is a word with several different meanings, but it is used in this book to mean a boundary or limit.

**prime number** (Lesson 14). A number which has exactly two factors, itself and one.

**product** (Lesson 4). The answer in a multiplication.

**quotient** (Lesson 5). The answer in a division.

**regroup** (Lesson 6). See “rename.”

**remainder** (Lesson 3). **1.** The answer in a subtraction; see difference. **2.** The leftover amount in a division; the part not divided.

**rename** (Lesson 6). **1.** In addition, regroup ten of one quantity for one of the group under the column heading to its immediate left. **2.** In subtraction, regroup one of a quantity for ten in the column to its immediate right; also called “exchange.”

**running total** (Lesson 12). An arithmetic operation where subtotals are found using two numbers at a time, and then another number is added to, subtracted from, used to multiply, or divided into the prior total; (often done with a calculator).

**Sieve of Eratosthenes** (Lesson 14). A hundreds square 10 across and 10 high naming the first 100 natural numbers and used to find prime numbers.

**subtrahend** (Lesson 8). The amount being subtracted, or the bottom number in a place value subtraction.

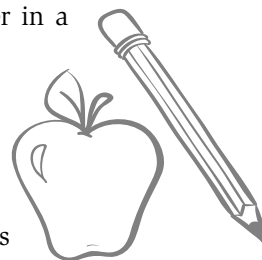
**sum** (Lesson 2). The answer in an addition.

**tolerance** (Lesson 11). The allowable amount of variation from a preset standard.

**trial dividend** (Lesson 10). Created by rounding the first two digits of a division’s dividend or partial dividend to the nearest 10 for use with the trial divisor.

**trial divisor** (Lesson 10). Created by rounding the divisor to the nearer 10, 100, or whatever and then using only the leftmost digit to estimate a partial quotient in conjunction with the partial dividend.

**within tolerance** (Lesson 11). Within acceptable limits; see “tolerance.”

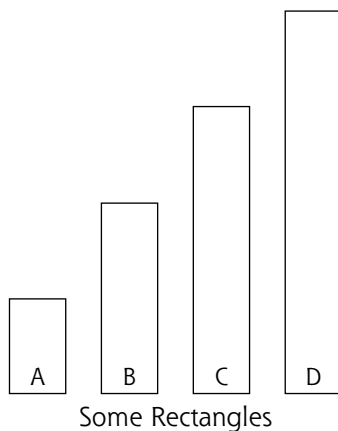


# LESSON 1

## Order and Magnitude

Most people think of numbers as a way of counting things, such as the number of students in a class or the number of lemon seeds in a glass of iced tea. Counting certainly is one way to use numbers. The numbers we count with are known as **cardinal numbers**. They have other names too, such as **counting numbers** and **natural numbers**. Totally distinct from the cardinal numbers, but obviously related to them, are the **ordinal numbers**. These are the numbers that are used to show position, that is 1st, 2nd, 3rd, . . . 25th, and so on. Some elementary math teachers will dwell upon these distinctions, but others will not. It is, however, a distinction that you should make sure that your child knows.

A much more important distinction exists in the meaning of numbers that some children are not taught, but are left to stumble upon, if they ever do. Rather than spelling it out immediately, I'd like you to consider the following diagram.



Suppose that we let the rectangle labeled “A” have a value of 1. Which rectangle would have a value of 2? You didn’t have to think about that for very long, did you? If “A” has the value of 1, then the next one, “B” would be two. It’s simple enough. Two comes after one; one comes before two.

Now, with that in mind, let’s assign the value 1 to rectangle “B.” Which rectangle would have the value 2? Hmm, this one may require a bit more thought. I’m certain that you’re not considering rectangle “A” for a multitude of reasons, which I shall not yet share with you. How about rectangle “C?” C is next in line. Just as “B” followed “A”, “C” follows “B.” Well, that might or might not look good to you, but I’m afraid that “C” won’t do. That’s because there’s more to the concept of numbers than just order or sequence. Two doesn’t just come after one. Its **magnitude** is twice that of one. Which rectangle follows “B” and is twice as big as “B?” That would be rectangle “D.” Therefore, if “B” is 1, “D” is 2.